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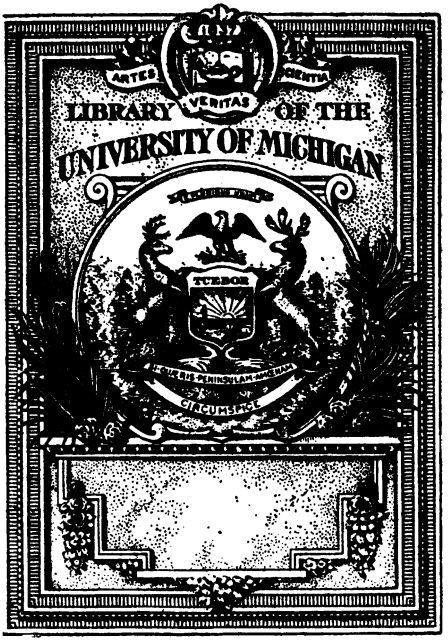

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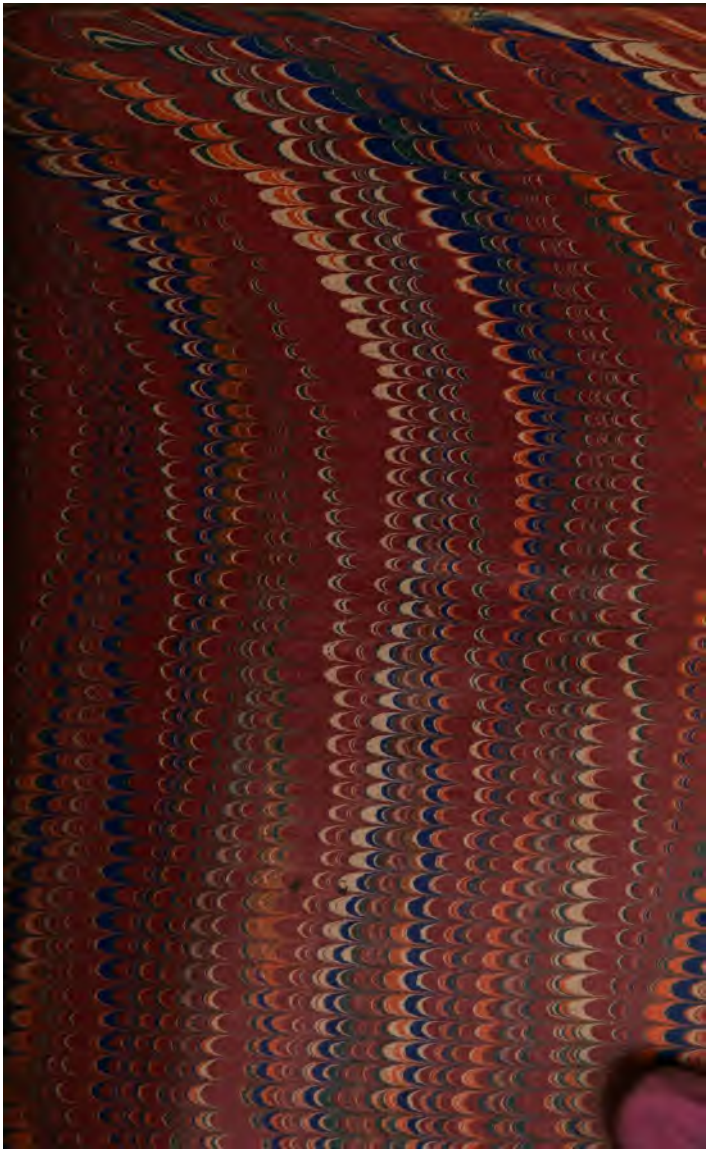

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TRIGONOMETRIA
PLANA ET SPHÆRICA
CUM SELECTIS
EX GEOMETRIA ET ASTRONOMIA
PROBLEMATIS.

Accedunt Sinuum Canones, & ex Euclide Propositiones magis necessariæ.

AUCTORE
R. P. (JACOBO) GOODEN
Societatis JESU, in Collegio Anglorum Leodii Matheseos Professore.

LEODII,
Apud J. F. BRONCART. Anno 1704.

Superiorum Permissu.

Prof. L. C. Karpinski
ex
4-30-1924

12 My 24 EHW

PRÆFATIO.

AD Tractatum hunc de Trigonometria conſcribendum impulêre tum ipſius ſcientiæ ſumma neceſſitas & præſtantia, tum multò magis infrequentia ſcriptorum, qui in explicandis Matheſeos rudimentis ſe Tyronum captui ſubmittant. Plerique enim vel brevitatis plus æquo ſtudioſi, Lectores ſuos non tam efficiunt peritos, quàm præſupponunt; vel perplexâ rerum varietate impares diſcentium animos obruunt & confundunt: ita ut difficile ſit quempiam invenire qui omnia Trigonometriæ præcepta clarè ſuccinctèque demonſtret. Hinc fit ut plerique Matheſeos Candidati hæreant in ipſo limine, vel librorum ſubſidio deſtituti, vel nimiâ eorundem mole deterriti.

Ut huic incommodo occurreretur, Opus hoc incipientium ingeniis ita attemperare conatus ſum, ut neque moles faſtidium pareret, nec brevitas obſcuritatem. Selegi proinde ea, quorum notitiam longa multorum experientia docuit neceſſariam; quæ ad finem propoſitum minùs ſunt idonea, utcunque ſcitu digna, penitùs relegavi; quæ verò talia ſunt ut juvare quidem poſſint, non tamen prorſus requirantur, vel obiter perſtrinxi, vel curavi ita à reliquis ſecernenda, ut neque officere poſſint incipienti, neque à proficiente deſiderari.

Trigonometriæ Planæ ſubjunxi problemata ſelecta, quibus præcipuam Practicæ Geometriæ partem ſum complexus; tum ut eſſent exemplorum loco, tum ut Tyrones præſenti laborum fructu recreatos ad magis

ardua confirmarent; idem quoque post Sphæricam absolutam eodem consilio præstiti, problematis illustrioribus ex Astronomia non sine delectu excerptis, iis præsertim quæ in Horographia, aliisque Matheseos partibus usui sunt futura.

Denique ut ad manum haberentur omnia, quæ scientiæ hujus Theoria vel Praxis exigit, selegi ex Euclide elementa maximè necessaria; quibus adjeci Sinuum & Tangentium Canones, cum numerorum Logarithmis, quales ad Geometricas operationes magis obvias requiruntur & sufficiunt. Quæ omnia conatus sum in talem ordinem redigere, ut sperem à quovis posse facilè percipi, & inoffenso pede percurri. Methodum autem quam in præcipuis demonstrationibus secutus sum, antiquâ longè commodiorem esse facilè intelliget, qui horum brevitatem cum aliorum prolixitate contulerit.

Notanda.

1. Elementa eo ordine citantur, quo habentur in selectis ad libri hujus calcem appositis.

2. In propositionibus citandis *p* significat ipsam propositionem, *c* caput, *b* hujus, *P* partem. Reliqua verborum compendia quivis per se intelliget.

3. Proportiones brevitatis causâ sic exprimo, A : B :: C : D. Lege, A est ad B, ut C ad D

4. Quando sermo est de Figura aliqua, quæ in ipso loco non apponitur; respicienda est Figura proximè præcedens, nisi aliud indicetur.

TRACTATUS DE TRIGONOMETRIA.

TRianguli *Reſolutio*, quæ Græco nomine *Trigonometria* dicitur, non aliud eſt quàm alicujus in triangulo propoſito lateris aut anguli inventio: & quia triangula alia ſunt plana ſeu rectilinea, alia ſphærica; hinc fit ut Trigonometria, pro objectorum diverſitate, in *Planam* & *Sphæricam* dividatur.

PARS PRIMA.

DE TRIGONOMETRIA PLANA.

PArtem hanc, quæ de Trigonometriâ planâ tota eſt; in ſex capita dividimus; quatuor priora totam de triangulis rectilineis doctrinam complectentur: in reliquis inſignem Trigonometriæ in quibuſvis dimenſionibus inveſtigandis uſum indicabimus.

CAPUT I.

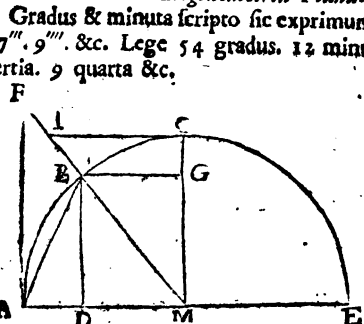
Præmittuntur aliqua ad Trigonometriam Planam.

DEFINITIONES.

1. CIrculi circumferentiam dividunt Mathematici in partes 360. quas vocant *Gradus*; unde ſemicircumferentia gradus continet 180. quadrans 90.

2. *Minutum* eſt 60a pars Gradûs: *Secundum* 60a pars minuti: *Tertium* 60a pars ſecundi, & ſic deinceps.

Gradus & minuta scripto sic exprimuntur: 54 gr. 12′. 34″. 17‴. 9⁗. &c. Lege 54 gradus. 12 minuta. 34 secunda. 17 tertia. 9 quarta &c.



3. Angulus aliquis (AMB) notus dicitur, quando cognoscuntur gradus arcûs (AB) subtensi, qui ex (M) anguli vertice, tanquam centro, describitur.

4. *Sinus rectus* (qui simpliciter vocatur sinus) anguli alicujus (AMB, vel BME) est recta (BD) ducta ab arcûs subtensi (AB, vel BE) uno extremo (B) perpendiculariter ad diametrum (AE) transeuntem per ejusdem arcûs alterum extremum (A vel E)

5. *Sinus versus* alicujus anguli (AMB) est diametri pars (AD) intercepta inter sinum rectum (BD) & arcum (AB) angulo subtensum.

6. Sinus anguli recti (AMC) seu *Sinus totus* est ipse circuli radius (CM) ut patet ex defin. 4.

7. *Tangens* alicujus anguli (AMB) est recta (AF) ab arcûs subtensi (AB) uno extremo (A) erecta perpendiculariter ad diametrum (AE) & producta donec occurrat rectæ (MF) ductæ à centro (M) per (B) alterum extremum ejusdem arcûs (AB)

8. Recta (MF) cui tangens sic occurrit, est *secans* ejusdem anguli (AMB)

Nota eosdem semper esse angulorum & arcuum subtensorum sinus, tangentes, & secantes.

9. *Complementum* arcûs vel anguli ad quadrantem vel angulum rectum, est ejusdem arcûs vel anguli differentia (sive per excessum, sive per defectum) à quadrante vel angulo recto. Hinc angulorum (AMC, BME) complementum est angulus (BMC) qui uni (AMC) additus, alteri (BME) subtractus relinquit angulum rectum (CMA, vel CME.) Similiter arcuum (AB, BE) complementum est arcus (CB) eorum differentia à quadrante (CA vel CE.)

Complementi CB, vel CMB sinus Rectus est BG, sinus Versus CG, Tangens CI, Secans MI.

10. Complementum arcûs vel anguli ad semicirculum vel duos rectos, est arcus vel angulus qui reliquus est ad semicirculum vel duos rectos.

In Trigonometria Plana vel citantur, vel præsupponuntur sequentes Euclidis Propositiones; quas proinde diligenter legerit oportet, ac intellexerit quisquis ad hanc Matheseos partem accedit.

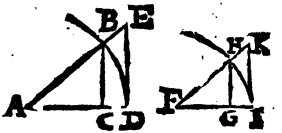
Ex Libro 1°. Definitiones & Axiomata. Prop. 1. 4. cum scholio. 5. 6. 8. 9. 10. 12. 13. 15. 22. 23. cum scholio. 27. 29. 32. cum 1. 4. & 5. Corol. 34. 35. 37. 38. 41. 47.

Ex Libro 3°. Definitiones omnes, & Prop. 3. 20. 21. 22.

Ex Libro 5°. Definitiones omnes, & Prop. 7. 12. 15. 16 cum 2 Corol. 17. 18.

Ex Libro 6°. Definitiones omnes, & Prop. 1. 2. 4 cum 1 & 2 Corol. 5. 6. 33.

PROPOSITIO I.



In circulis quibuscunque eadem est ratio radiorum (AB, FH) *ad sinus* (BC, HG) *tangentes*, (ED, KI) *& secantes* (AE, FK) *similium arcuum* (BD, HI) *& angulorum æqualium* (A & F)

Demonstr. In triangulis ABC, FGH anguli A & F sunt (*ex suppos.*) æquales, & anguli C & G (*per* 4 *def. h*) recti, adeoque æquales. Ergo (*per* 2 *Corol. p.* 4. *l.* 6.) AB: FH:: BC: HG.

Eodem modo ostendas tangentes ED, KI, & secantes AE, FK esse inter se, ut sunt radii AD, FI.

PROPOSITIO II.

De inventione Sinuum, Tangentium & Secantium.

Invenire alicujus anguli BAC sinum BC, tangentem ED, vel secantem AE, est determinare proportionem quam habent ad suum sinum totum, seu radium circuli AB; id est,

posito quòd AB semidiameter dividatur in quotcunque lubuerit partes, vg. 10000000, ostendere quot hujusmodi partes contineat sinus, tangens, vel secans cujusvis anguli.

Sinuum inquisitio prolixa est & operosa, neque ad institutum nostrum necessaria, cùm passim prostent tabulæ sinuum proportionem exhibentes. Modus quorumvis arcuum & angulorum sinus investigandi videri potest apud P. Tacquet, Geometriæ Pract. libro 1. cap. 2.

Sinubus semel notis, facilè inveniuntur tangentes & secantes iis correspondentes; Nam in triangulis ABC, AED anguli C & D (*per* 4. & 7. *def. h.*) sunt recti, & angulus A communis: Ergo (*per* 2. *Corol. p.* 4. *l.* 6.) Latus AC quod (*per* 34. *l.* 1.) æquale est sinui complementi (ut patebit comparanti DM cum BG in Fig. 1.) est ad BC sinum anguli A; ut radius AD, ad ED tangentem anguli A. Et AC sinus complementi, ad AD sinum totum, ut AB sinus totus, ad AE secantem anguli A.

Corollarium.

HIs propositionibus nititur constructio tabularum, quarum per Mathesim universam usus est frequentissimus, & vocantur *Sinuum*, *Tangentium* & *Secantium Canones* seu *Tabulæ*, quia sinus &c. pro gradibus singulis & minutis computatos exhibent, id est, numeris exprimunt sinuum &c. proportionem ad suam semidiametrum in partes 10000000 divisam. Omittuntur autem sinus &c. minutis secundis & tertiis debiti, tum ne moles in immensum cresceret: tum quia semidiametro in tot partes dissectâ, istiusmodi minutiæ sine dispendio negliguntur. Quòd siquando calculus accuratior desideretur, sinus &c. quibuscunque minutis debitos methodo infrà præscribendâ facilè obtinebis.

Notandum porrò tabulas in duplici esse genere; aliæ vocantur *Naturales* quæ veram proportionem explicant numeris Geometricè proportionalibus: aliæ *Artificiales* quæ naturalium numerorum loco substituunt Logarithmos, seu numeros Arithmeticè proportionales, de quorum inventione & usu, quia alibi dictum est, nil adjicio.

PRO-

PROPOSITIO III.

Minutis quibuſcunque datis exhibere ſinus ipſis debitos: & converſo notis ſinubus, minuta ipſis correſpondentia invenire.

Requiratur vg. ſinus 30 gr. 5′. 8″. Invento in tabulis, ſinu proximè majore, ſeu ſinu 30 gr. 6′. qui eſt 5015108 & proximè minore, 30 gr. 5′, 5012591; quæratur (*per ſubtractionem*) eorum differentia 2517. Dein fiat ut unum minutum ſeu 60″ ad 2517, ſic 8″ ad aliud. Erit quartus terminus, Fractione ſpretâ, 335; qui additus ſinui 30 gr. 5′ dat 5012926 ſinum quæſitum gr. 30. 5′. 8″.

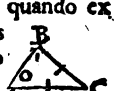
E converſo. Si dato ſinu aliquo, vg. 2985411, quærantur gradus & minuta ipſi correſpondentia; niſi in tabulis compareat ipſiſſimus ſinus datus, inde deſumantur ſinus proximè minor viz: 17 gr. 22′. ſeu 2984856, & proximè major 2987632. gr. 17 23′. Quorum differentia eſt 2776. Deinde ex ſinu dato 2985411 ſubtrahatur proximè minor 2984856 ut habeatur exceſſus 555. Jam ſi fiat ut 2776 differentia inter ſinus proximos dato, ad 60″: ſic 555 exceſſus ſinûs dati ſuprà ſinum proximè minorem, ad aliud; quartus terminus 12, indicabit quot minuta ſecunda illi exceſſui correſpondeant.

Eodem modo reperiuntur ſinus quibuſvis minûtis debiti; & quod dictum eſt de ſinubus, poteſt ſimiliter tangentibus & ſecantibus explorandis applicari.

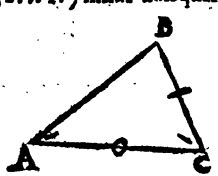
Notanda.

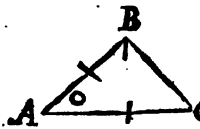
1. Ad trianguli cujuſcunque analyſim ſeu reſolutionem prærequiruntur tria cognita, viz: duo anguli cum uno latere, vel latus unum cum duobus angulis, vel tria latera, vel demum in ſphæricis tres anguli. Dixi *in Sphæricis*, quia in triangulis planis ex ſolis angulis determinari poteſt proportio laterum, non tamen (*ut patet ex 4. l. 6.*) eorum quantitas abſoluta. Termini dati ſeu cogniti notari ſolent lineolâ, quæſitus cyphrâ deſignatur, ut videre eſt *in Fig. ſequenti.*

2. In triangulo aliquo dicitur dari *Oppoſitio*, quando ex terminis notis duo opponuntur inter ſe, & tertius opponitur termino quæſito. Unde in triangulo ABC datur oppoſitio, nam latus notum AC oppo nitur angulo noto B, & notum latus BC opponitur angulo quæſito A.

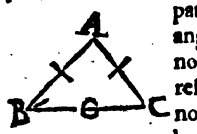


3. Quia omnes anguli cujuſcunque trianguli rectilinei (*per 32. l. 1.*) ſimul adæquant duos rectos, hoc ipſo quòd cognoſcantur quivis duo anguli A & C, innoteſcet tertius B, ſubtrahendo ſummam notam angulorum A & C ex duobus rectis, ſeu gradibus 180. ſint vg. angulus A 38 gr. 19′, angulus C 63 gr. 27′. Erit eorum ſumma 101 gr. 46′; quæ ſubtracta ex gr 180 relinquit 78 gr. 14′. quantitatem tertii anguli B. Hinc 1.° Quotieſcunque dantur duo quivis anguli A & C cum quovis latere BC, ſemper habetur oppoſitio; nam ex duobus angulis datis A & C invenitur angulus B oppoſitus lateri quæſito AC. 2.° Dato angulo B, ut inveniatur alter A, ſufficit quòd detur oppoſitio reſpectu tertii C; nam C invento, etiam A notus erit.





Ex dictis ſequitur oppoſitionem tantùm deeſſe quando dantur tria latera ſine angulo, vel duo latera cum uno angulo; qui ſi contineatur inter latera data (*ut in Prop. 3. cap. 3.*), patet quòd non detur oppoſitio: ſi verò ſit angulus B tantùm adjacens alicui lateri noto AB, patet quòd habeatur oppoſitio reſpectu alterius anguli adjacentis C; quo noto, ſcitur etiam angulus A; adeoque habetur oppoſitio reſpectu tertii lateris BC.



4. Quæſtiones Trigonometricæ reſolvuntur per Regulam Auream; adeoque termini ſunt juxta tenorem iſtius Regulæ diſponendi: ita ſcilicet ut primus ſit ejuſdem rationis cum tertio, & ſecundus ejuſdem cum quarto quæſito.

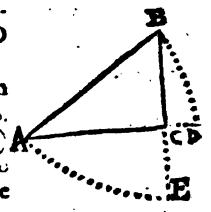
CAPUT II.

De Analyſi Trianguli rectanguli.

PROPOSITIO I.

In triangulo quovis rectangulo unumquodque latus eſt ſinus anguli ſibi oppoſiti.

Aſſumatur pro radio hypotenuſa AB; & centris A & B

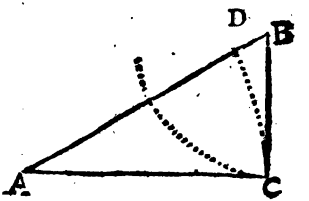


deſcribantur arcus AE, BD, quibus occurrant latera AC, BC producta in D & E.

Patet [*ex Conſtr.*] latus AB eſſe ſinum totum, nam eſt radius utriuſque arcûs. Deinde quia angulus C eſt (*ex ſuppoſ.*) rectus, latus BC ductum à B uno extremo arcûs ſubtenſi BD, eſt perpendiculare diametro AD tranſeunti per D alterum extremum ejuſdem arcûs: ergo [*per 4. def. h*] BC eſt ſinus anguli oppoſiti A reſpectu ſinûs totius AB.

Similiter demonſtres AC eſſe ſinum anguli oppoſiti B, reſpectu ejuſdem radii BA ſeu AB.

PROPOSITIO II.



Si in quovis triangulo rectangulo [ABC.] *quodcunque latus* [AC vel BC] *circa angulum rectum* [C] *aſſumatur pro ſinu toto; erit alterum latus tangens anguli ſibi oppoſiti, reſpectu ejuſdem ſinûs totius.*

Centro A, radio AC, ducatur arcus DC ſubtenſus angulo A.

Quia angulus C eſt [*ex ſuppoſ.*] rectus, latus BC eſt perpendiculare ad diametrum AC, & erigitur à C uno extremo arcus ſubtenſi DC, donec in B concurrit cum AB ductâ ex centro A per D alterum extremum ejuſdem arcûs ſubtenſi DC: ergo [*per 7. def. h*] BC eſt tangens anguli ſibi oppoſiti A, reſpectu radii ſeu ſinûs totius AC. Eodem modo oſtendas AC eſſe tangentem anguli B, reſpectu radii BC.

Corollarium.

Ex propoſitionibus explicatis duæ eliciuntur regulæ, quarum ope quodvis triangulum rectangulum facilè reſolvetur. Prima & principalis vocatur *Regula ſinuum*, quia dependet à proportione, ſeu potiùs identitate, inter trianguli latera & ſinus angulorum oppoſitorum; vocatur etiam *Regula Oppo-*

ſitionis, quia locum habet quotieſcunque termini propoſiti inter ſe opponuntur. Secunda eſt *Regula Tangentium*, ita dicta quòd nitatur proportione quam habent latera circa angulum rectum ad tangentes angulorum oppoſitorum. Hæc Regula adhibetur, quoties non intervenit oppoſitio.

Regula Sinuum.

In omni triangulo rectangulo
Ut quodvis latus, eſt ad ſinum anguli ſibi oppoſiti:
Sic aliud quodvis latus, eſt ad ſinum anguli ſibi oppoſiti.

Et è contrà,

Ut ſinus cujuſvis anguli, ad latus ſibi oppoſitum:
Sic ſinus cujuſvis alterius anguli, ad latus ſibi oppoſitum.

Patet ex 1. *p*: *h*. ubi oſtenſum eſt unumquodque latus eſſe ſinum anguli ſibi oppoſiti.

Regula Tangentium.

In omni triangulo rectangulo
Ut latus unum circa angulum rectum, ad ſinum totum:
Sic aliud latus circa angulum rectum, ad tangentem anguli ſibi oppoſiti.

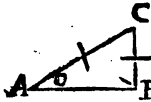
Et è contrà,

Ut ſinus totus, ad latus unum circa angulum rectum:
Sic tangens anguli, qui alteri lateri opponitur, ad aliud latus.

Patet ex 2. *p*. *h*, ubi demonſtratum eſt utrumlibet latus circa angulum rectum eſſe tangentem anguli ſibi oppoſiti reſpectu alterius lateris circa angulum rectum aſſumpti in ſinum totum.

PROPOSITIO III.

Reſolvere triangulum rectangulum, in quo datur oppoſitio.



1.o In triangulo ABC nota ſint latus AC 100 paſſuum, latus BC 50 paſſuum, & angulus rectus B: quæratur autem angulus acutus A.

Quia datur oppoſitio, adhibenda eſt regula ſinuum, & quia quæritur angulus, termini ſunt ita diſponendi, ut quarto

loco veniat angulus quæsitus A, adeoque secundo loco terminus ejusdem rationis, viz: angulus notus B. Unde primus terminus erit latus AC oppositum angulo noto B, & tertius latus BC oppositum angulo quæsito A. Hæc terminorum dispositio semper est observanda, quando per regulam sinuum quæritur angulus. Fiat itaque

Ut latus AC, 100 *passuum, oppositum angulo noto* B,
Ad sinum anguli noti B, *viz:* 10000000:
Sic latus BC, 50 *passuum oppositum angulo quæsito* A,
Ad sinum anguli quæsiti A.

Peractâ operatione juxta Regulam Proportionis, id est; multiplicato secundo termino [10000000] per tertium [50] & producto [500000000] diviso per primum [100;] quotiens [5000000] erit quartus terminus & sinus anguli quæsiti, qui in tabulis sinuum inventus ostendit angulum A continere gradus 30.

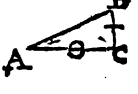
2.° In triangulo EFD dentur angulus D rectus, latus EF 800 pedum, & angulus F 54 gr. 20′. quæratur verò latus ED.

Quia datur oppositio, solutio quærenda est per regulam sinuum; & quia quæritur latus, termini hoc ordine disponentur.

Ut sinus anguli recti D, *seu sinus totus* 10000000
Ad latus oppositum EF 800 *pedum:*
Sic sinus anguli dati F 8124229
Ad latus oppositum & quæsitum ED

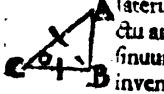
Factâ multiplicatione tertii termini per secundum, & divisione producti per primum; invenitur quartus terminus $649\frac{9383200}{10000000}$ Unde concludes latus DE continere pedes $649\frac{9}{10}$ circiter.

3.° Datis angulis A & C cum latere BC, si quæratur latus AC neutri angulo dato oppositum; 1.° ex angulis notis A & C inveniatur (*ut in Not: 3. c. 1.*) angulus tertius B, quo noto, habetur oppositio; unde 2.° per regulam sinuum invenietur latus AC, ut in casu præcedenti.



4.° Notis lateribus AC, BC, cum angulo recto B eorum

uni oppoſito; oporteat invenire angulum C neutri notorum laterum oppoſitum. Quia datur oppoſitio reſpectu anguli A non quæſiti, exploretur per regulam ſinuum, (*ut in caſu* 1. *b*) angulus A; nam illo invento, [*per Not.* 3. *c.* 1.] notus erit angulus quæſitus C.



Tres ſuperſunt caſus, in quibus non datur oppoſitio ſufficiens, ut per regulam oppoſitionis obtineatur ſolutio, ſaltem immediatè, & unicâ operatione. Horum caſuum ſolutionem dabunt Propoſitiones ſequentes.

PROPOSITIO IV.

Datis lateribus (AB, BC) *circa angulum rectum* (B) *invenire quemcunque angulum acutum* (C)

Caſus hic ſolvitur per regulam tangentiam, juxta quam ſunt ſemper ita diſponendi termini, ut primus ſit latus [BC] adjacens angulo quæſito (C;) ſecundus ſinus, totus, ſeu ſinus anguli (B) recti; tertius B latus [BA] oppoſitum angulo (C) quæſito; quartus angulus (C) quæſitus. Sit BC 6000 pedum, BA 10000 pedum. Termini ſic ſtabunt.

Ut latus BC 6000 *pedum*, *Ad ſinum totum* 10000000:
Sic latus AB 10000 *pedum*, *Ad tangentem anguli oppoſiti* C

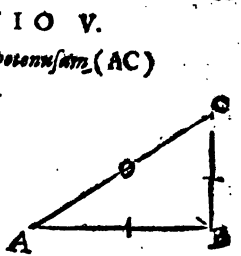
Operare ex præſcripto Regulæ aureæ, & deprehendes quartum terminum $16666666\frac{4}{6}$ tangentem anguli quæſiti C; cui in tabulis tangentium quæſitæ proximus invenitur numerus 16664748 tangens anguli 59 gr. 2′, adeoque angulus quæſitus C eſt 59 gr. 2′. proximè.

Nota per analyſim trigonometricam rarò reperiri ipſiſſimos ſinus & tangentes, qui in tabulis habentur; accipi proinde debent quæſitis proximi, neglectâ differentiâ, quæ minoro ſemper eſt minor.

PROPOSITIO V.

Iisdem datis, invenire hypotenusam (AC)

Ad casûs hujus solutionem utraque regula adhibetur.

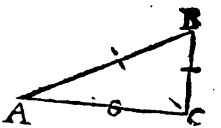


1.o *Per Regulam tangentium, ut in præcedente*, investigatur aliquis angulus acutus (C) quo noto, datur oppositio. Unde

2.o *Per Regulam oppositionis, ut in Prop.* 1. invenitur hypotenusa quæsita AC.

PROPOSITIO VI.

Datâ hypotenusâ (AB) *cum aliquo latere* (CB) *circa angulum rectum* (C) *invenire aliud latus* (AC.)



Solvitur casus hic per *Regulam sinuum* bis adhibitam.

1.o Quæritur angulus A oppositus lateri noto BC & simul invenitur angulus B oppositus lateri quæsito AC.

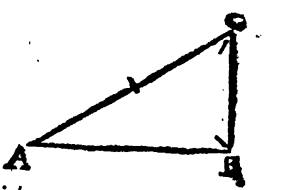
2.o Ex angulis B & C & latere dato AB, notum fit latus quæsitum AC.

PROPOSITIO VII.

Triangulum resolvere per Logarithmos.

Trianguli resolutio per Logarithmos in eo solo discrepat à praxi hactenus explicatâ, quòd multiplicationis & divisionis loco requirantur additio & subtractio; ut in exemplo constabit.

Sit in triangulo ABC angulus B rectus, angulus C 60 gr. & latus AC pedum 1000, inquiratur autem latus AB



Quia datur oppositio, termini (*per* 1. *b*) sic disponentur.

Ut Anguli recti B *sinus Logarithm.* 10000000

Ad Lateris oppositi AC *Logarith.* 30000000
Sic anguli C *sinus Logarithmicus* 99375306
Ad Logarithmum lateris quæsiti AB.

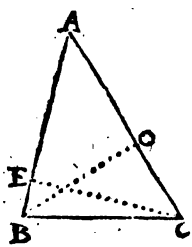
Secundi & tertii termini Logarithmis in unam summam (129375306) per Additionem collectis, subtrahe Logarithmum primi termini; residuum (29375306) est Logarithmus quarti termini, eique in tabulis invento correspondet numerus 866. Est igitur latus AB pedum 866. Idem est Logarithmorum usus, quando per regulam tangentium instituitur operatio.

Nota Logarithmos arcuum & angulorum quæri debere in tabulis Logarithmicis sinuum vel tangentium; Logarithmos verò lateribus rectilineis correspondentes inveniri in tabula Logarithmica numerorum.

CAPUT III.

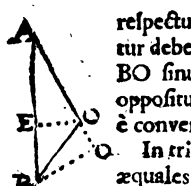
De Analysi trianguli obliquanguli.

PROPOSITIO I.



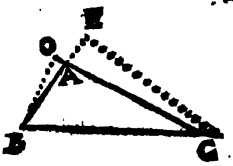
Trianguli cujusvis rectilinei latera sunt inter se ut sinus angulorum oppositorum; & è converso.

Proponantur triangula ABC. Ab extremis alicujus lateris BC ducantur perpendicula BO, CE ad latera opposita AC, AB, si opus fuerit producta. Erunt in triangulis rectangulis BOC, CEB, (*per p. 1. c. 2.*) Latera BO, CE sinus angulorum sibi oppositorum C & B, idque respectu ejusdem sinûs totius BC. Ostendi igitur debet latus AB oppositum angulo C, esse ad BO sinum ejusdem anguli C, sicut latus AC oppositum angulo B ad CE sinum anguli B: & è converso.



In triangulis AEC, AOB, anguli E & O sunt æquales, utpote (*ex constr.*) recti: & angulus A vel communis est ut *Fig.* 1. & 2. vel (*per* 15. *l.* 1.) utro-

utrobique æqualis, ut in *Fig.* 3. Ergo (*per* 2. *Corol. p.* 4. *l.* 6.) AB: AC :: BO : CE, & (*permutando*) AB : BO :: AC : CE. Ergo (quia similis est in reliqua laterum si nuumque comparatione demonstratio) in omni triangulo obliquangulo



eadem est ratio inter latera & sinus angulorum oppositorum.

Observari hic debet quòd proportio hæc æqualiter subsistat, sive omnes anguli sint acuti, sive aliquis sit obtusus, qualis est angulus BCA *in Fig* 2.

Corollarium.

Ex hac propositione deducitur pro resolutione trianguli obliquanguli sequens generalis.

REGULA SINUUM SEU OPPOSITIONIS.

In omni triangulo obliquangulo
Ut quodvis latus est, ad sinum anguli sibi oppositi:
Sic quodvis aliud latus est, ad sinum anguli sibi oppositi.

Et è contrà,

Ut sinus cujusvis anguli, ad latus sibi oppositum:
Sic sinus cujusvis alterius anguli, ad latus sibi oppositum.

Notanda.

1. Nomine trianguli obliquanguli hîc intelligimus omne triangulum quod vel non habet, vel nescitur habere aliquem angulum rectum.

2. Diligenter recolenda sunt notanda cap. 1. & quæ dicta sunt Prop. 3. cap. 2. de resolutione trianguli rectanguli habentis oppositionem; nam ista omnia triangulis obliquangulis possunt eodem modo applicari.

PROPOSITIO II.

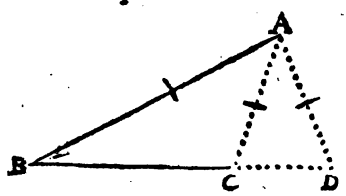
Resolvere triangulum obliquangulum, in quo datur Oppositio.

Quia (ut ostendimus) Regula Sinuum etiam in obliquangulis locum habet, methodus pro rectangulis superiùs (*p.* 3

c. 2.) tradita, est etiam in obliquangulis observanda, quoties intervenit Oppositio; quæ extra duos casus, semper datur sufficienter; ut per regulam sinuum, saltem bis adhibitam, eruatur quæsitum. Satis proinde fuerit duos istos casus attingere, in quibus oppositio desideratur. Priùs tamen

Nota 1.° quòd si detur aliquis angulus obtusus, accipi pro eo debeat in operatione, sinus complementi ad duos rectos: vg. Si notus sit angulus 137. gr. 15′; quæri debet in tabulis & in operatione adhiberi sinus 42 gr. 45′.

Nota 2.° quòd si dentur duo latera AB, AC cum angulo adjacente B, casus hic, tametsi interveniat oppositio, determinatam solutionem non admittat, nisi aliunde cognoscatur species anguli C. Nam, quia latus AB & angulus B sunt communia triangulis ABC, ABD, & latera AC, AD possunt esse æqualia; hæc non sufficiunt ad determinandum utrùm tertium latus sit BC, an verò BD; nec quales sint reliqui anguli.



Quòd si aliunde scias speciem anguli C, difficultas evanescit; nam in triangulo minore ABC semper est obtusus angulus C, in majore ADB semper est acutus D. quod sic ostenditur.

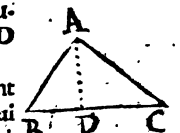
In triangulo ACD latera AC, AD sunt (*ex suppos.*) æqualia: ergo (*per* 5. *l.* 1.) æquantur anguli ACD, ADC: ergo, cùm simul sint (*per* 32. *l.* 1.) minores duobus rectis, singuli sunt acuti: ergo angulus D est acutus; &, quia ACD est etiam acutus, angulus ACB (*per* 13. *l.* 1.) est obtusus.

PROPOSITIO III.

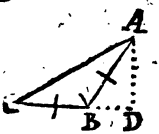
Datis duobus lateribus (AB, BC) *cum angulo comprehenso* (B) *invenire tertium latus* (AC) *& reliquos angulos* (A & C)

Ab angulo aliquo ignoto A dimitte perpendiculum AD

ad latus oppositum BC, si opus sit, productum. Sic fient duo triangula BAD, CAD rectangula in D.



Quoniam in triangulo ABD, noti sunt anguli, D (*ex constr.*) rectus, & ABD, qui vel idem est cum dato ABC, vel (*per 13. l. 1.*) dati complementum ad duos rectos; notus est etiam tertius angulus BAD. Ex omnibus angulis & latere dato AB (*per regulam sinuum*) inquirantur latera AD & BD.



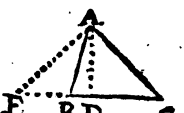
Invento BD, scitur etiam DC: ergo in triangulo ACD habentur latera duo AD, CD circa angulum D (*ex constr.*) rectum. Ex quibus (*per regulam tangentium*) noti fiunt anguli ACD, & CAD; & ex his omnibus (*per regulam sinuum*) reperitur latus AC.

Nota 1.° quòd si angulus datus ABC sit obtusus, accipi debeat in prima operatione angulus ABD ejus complementum ad duos rectos.

Nota 2.° quòd si inveniatur latus BD minus latere BC dato, perpendiculum cadat intrà; aliàs extra triangulum. Hoc diligenter est observandum ut constet utrùm additione, an subtractione opus sit, ut ex inventis eliciantur quæsita.

LEMMA I.

Si ad trianguli (ABC) *latus maximum* (BC) *ab angulo opposito* (A) *ducatur perpendiculum* (AD) *cadet intra triangulum.*

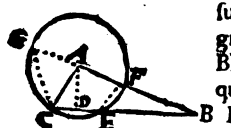


Si neges, cadat extra ad E; erit angulus E rectus: ergo (*per 47. l. 1.*) quadratum AC æquatur quadratis AE, EC adeoque majus est quadrato BC: ergo linea AC est major lineâ BC. quod est absurdum, quia BC supponitur latus maximum trianguli ABC.

LEMMA II.

Si ad trianguli (ABC) *latus maximum* (BC) *demittatur perpendiculum* (AD,) *erit basis* (BC) *ad aggregatum reliquorum laterum* (AB, AC:) *ut differentia horum laterum, ad differentiam segmentorum baseos.*

Centro A, intervallo minimi lateris AC, describatur circulus, cui occurrat in G, latus BA productum, quia æqualia sunt AC, AG, AF, patet BG esse aggregatum laterum AB, AC, & lineam BF esse eorundem differentiam. Item quia (*per* 3. *l.* 3.) æqualia sunt ED, DC, patet BE esse differentiam inter baseos segmenta BD, DC. Ostendi igitur debet quòd sit BC : BG :: BF : BE. Junge puncta G, C, & F, E.

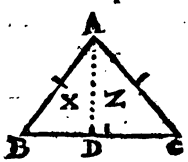


In triangulis GBC, EBF angulus B est communis, & angulus G cum FEC (*per* 22. *l.* 3.) æquatur duobus rectis. Sed FEC cum FEB (*per* 13. *l.* 1) facit duos rectos: ergo G & FEB sunt æquales: ergo (*per* 2. *Corol. p.* 4. *l.* 6.) BC: BG:: BF: BE.

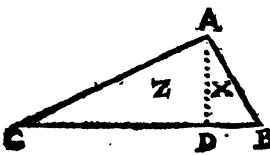
PROPOSITIO IV.

Datis tribus lateribus investigare angulos.

1.o Si aliqua duo latera AB, AC sint æqualia, ab angulo A inter illa comprehenso, ad basim BC bisectam in D, ducatur perpendiculum AD. Sic fient duo triangula X & Z sibi mutuò æquilatera, ut patet: & quia AB, AC ponuntur æqualia, anguli oppositi ADB, ADC (*per* 8. *l.* 1) sunt æquales, adeoque recti: ergo in triangulo X cognoscuntur latera AB & BD dimidium dati BC, cum angulo recto D. Ex his, (*per regulam sinuum*) explorantur anguli BAD, adeoque & ABD, quibus (*per* 8. *l.* 1.) æquantur anguli CAD, ACD



2.o Si latera omnia sint inæqualia, vg. BC pedum 21, AB 13, AC 20. demisso ab angulo A ad latus maximum BC, perpendiculo AD; erit ut basis BC (seu 21) ad aggregatum laterum AB, AC [seu 33:] sic horum laterum differentia (seu 7) ad differentiam segmentorum BD, DC. Ductis igitur 7 in 33, & producto 231 diviso

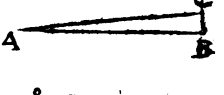


per 21, quotientem 11 aufer à basi BC 21. Residui dimidium 5 erit segmentum minus DB, adeoque segmentum majus D est 16. Ergo in triangulo Z nota sunt latera AC, DC, cum angulo recto ADC, & in triangulo X latera AB, BD cum angulo recto ADB; ex his (*per regulam sinuum*) innotescent anguli A, B, C.

Nota ea omnia, quæ [*Prop. 7. cap. 2.*] *dicta sunt de usu Logarithmorum in resolutione trianguli rectanguli, esse etiam de obliquangulis intelligenda.*

Hic etiam notari debet quòd si, multiplicatis inter se secundo & tertio termino, prodeat numerus minor primo termino; quæsitum sit Fractio, cujus numerator est productus inventus, denominator verò primus terminus.

Exemplum. Detur in triangulo ABC, angulus B rectus, ejusque sinus 10000000; latus AC 8 pedum; & angulus A 5 gr. 44′ 21″, cujus sinus est 1000000; quæratur autem latus BC. Secundo termino 8, ducto in tertium 1000000, habetur Productus 8000000 minor primo termino 10000000; unde latus quæsitum BC est minus pede, seu Fractio pedis quæ sic exprimitur $\frac{8000000}{10000000}$ vel sic $\frac{8}{10}$ assumpto nimirum in denominatorem primo termino, in numeratorem Producto invento.



Si, dum per Logarithmos instituitur operatio, summa secundi & tertii termini sit minor primo; erit quartus terminus Fractio, cujus denominator est numerus correspondens Logarithmo primi termini, numerator verò numerus correspondens summæ per secundi & tertii termini additionem inventæ.

CAPUT IV.

De analysi triangulorum per scalam & quadrantem.

PROPOSITIO I.

Constructio Scala Geometrica.

Nomine *Scala* hîc intelligitur regula quævis in 10 partes

æquales divisa secundùm longitudinem; lineis 0. 1. 2. 3. &c. à latere ad latus ductis; & in 10 partes æquales secundùm latitudinem, lineis ductis ab extremo ad extremum. Subdividitur etiam suprema decima pars in alias 10 partes æquales, per lineas obliquas & parallelas, 0. 2. 4. 6. &c. Demum lineis longioribus intermediis adscribuntur in regulæ extremo notæ, 2, 4, 6 &c. & perfecta est scala.

Tota scalæ longitudo accipitur pro 1000 partibus, unde pro pluries 1000 partibus est pluries repetenda. Pro centenis assignantur novem decimæ partes scalæ. Pro decadibus relinquitur una decima pars in suas decades distributa. Numerantur unitates in lineis quæ unitatum notis designantur.

Ex: gr: Ko exhibet partes 300: K8 partes 380 : RT partes 354 : *ei* unam partem : *xz* duas : *sn* tres : *lm* septem &c.

Quòd partes centenæ & decades rectè numerentur, patet ex constructione scalæ; quòd autem rectè numerentur unitates, sic ostendo.

In triangulo *Io*, *o*, *o* ducitur latus *ie* parallelum [*ex constr.*] lateri *Io*, *o*: ergo (*per* 1. *Corol. p.* 4. *l.* 6.) *ie* : *Io*, *o* :: *oe* : *oo*. sed *oe* est (ex constr.) decima pars rectæ *oo* : ergo *ie* est decima pars rectæ *Io*, *o*. Eodem modo probatur *xz* esse $\frac{2}{10}$, *ns* $\frac{3}{10}$ *lm* $\frac{7}{10}$ ejusdem rectæ *Io*; & sic de cæteris.

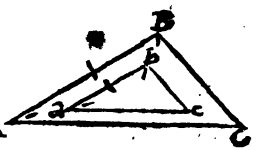
PROPOSITIO II.

Resolvere quodcunque triangulum per Scalam & Quadrantem.

Usus Scalæ & Quadrantis in triangulorum resolutione ad quatuor casus reducitur; nec refert utrùm triangulum propositum sit rectangulum an verò obliquangulum; nam eadem in utroque methodus observatur.

Primus Casus.

Ex aliquo latere (AB) *& duobus angulis* (A & B) *reliqua latera invenire.*



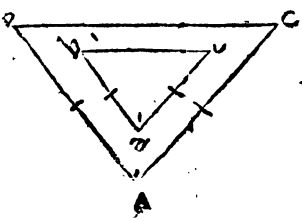
Contineat latus datum AB pedes 143. Angulus A 36 gr. 9′. Angulus B 99 gr. 12′.

Desume ex scala lineam *a b* dato lateri AB similem, id est, continentem tot scalæ partes quot pedes continet AB. Dein lineâ *a b* in chartam translatâ (*per schol.* 23. *l.* 1.) effice angulos *a* & *b* æquales datis A & B, & perfice triangulum *abc* quod (*per* 2. *Corol. p.* 4. *l.* 1.) erit simile dato ABC. ergo (*per* 4. *l.* 6.) *ab* : AB :: *bc* : BC, & *ab* : AB :: *ac* : AC. ergo etiam BC, AC tot pedes continent, quot scalæ partes continent *bc*, *ac*. Has igitur circino exceptas scalæ admove, & innotescent latera BC, AC. viz: BC 120 pedum : AC 201 pedum.

Nota perinde esse quinam duo anguli dentur; nam ex quibusvis duobus datis tertius cognoscitur.

Secundus Casus.

Ex duobus lateribus [AB, AC] *& angulo contento* [A] *invenire aliud latus* [BC] *& reliquos angulos.*



Accipiantur ex scala *ab*, *ac* similes datis AB, AC; factoque (*per schol.*

23. *l.* 1.) angulo *bac* æquali dato BAC, jungantur puncta *b* & *c*. Erunt (*per* 6 *l.* 6.) triangula *abc*, ABC similia; adeoque *ab*: AB:: *bc*: BC. Ergo *bc* scalæ applicato invenietur latus BC quæsitum.

Deinde quia latera triangulorum *abc*, ABC sunt similia, anguli B, *b*, & C, *c*, oppositi similibus lateribus AC, *ac*, & AB, *ab* (*per* 5. *l.* 6.) sunt æquales. Inventis igitur [*per schol. p.* 23. *l.* 1.] angulis *b* & *c*, noti erunt anguli quæsiti B & C.

Tertius Casus.

Datis omnibus lateribus invenire angulos.

Si ex scala demas tres lineas similes lateribus datis, triangulum ex eis [*per* 22. *l.* 1.] constitutum, erit (*per* 5. *l.* 6.) æquiangulum triangulo dato. Unde si trianguli istius angulos explores [*per schol. p.* 23. *l.* 1.] inde cognoscentur anguli quæsiti.

Quartus Casus.

Datis duobus lateribus cum aliquo angulo adjacente, reliqua invenire.

Hic est casus ille indeterminatus de quo dictum est Prop. 2. cap. 3. Vide Figuram istic positam, cui si ex scalæ partibus similem confeceris, erunt (*per* 5. *l.* 6.) triangula æquiangula, & latera proportionalia: adeoque tertium latus BC vel BD, & anguli non dati, modo superiùs tradito, noti fient. Casus hic infrequens est, & ad trianguli analysim ferè inutilis; unde non est cur illi diutiùs immoremur.

CAPUT V.

Longitudinum & Altitudinum Dimensio.

Trigonometriæ Planæ hactenùs explicatæ ut usus constet, praxisque evadat facilior, problemata Geometrica adjungere visum est; pauca quidem pro rerum varietate, talia tamen, ut præcipuam scientiæ illius partem complectantur.

Distan-

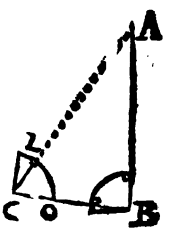
Distantiarum seu Longitudinum Horizontalium investigatio.

PROBLEMA I.

Metiri distantiam ex una tantùm parte accessibilem.

PRIMUS MODUS.

Sit talis distantia BA, latitudo Fluminis vg. accessibilis tantùm in B. eligatur aliqua alia statio C, & mensuretur mechanicè stationum intervallum CB, quod oportet modiocre esse respectu distantiæ quæsitæ BA. Dein quadrante explorentur anguli C & B; ex quibus & latere CB notis, invenietur (*per. 2. c. 3. h.*) latus BA Fluvii latitudo quæsita.

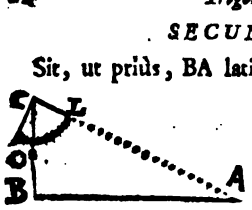


Nota 1.° Ad intervallum aliquod mechanicè mensurandum adhiberi posse vel catenulas ferreas, vel decempedam ligneam, vel aliud ejusmodi instrumentum. Inter mensurandum autem curare debet mensor ut pergat in directum, sequendo semper eandem lineam rectam; ad quod juvabunt duo baculi modico intervallo distantes, erecti vg. in C & O, à quibus ita retrocedat mensor, ut oculus ejus constanter maneat in eadem linea cum utroque baculo.

Nota 2.° Hujusmodi trianguli, quod scil: vel horizontale est, vel ad horizontem inclinatur, angulos exquiri posse ope quadrantis vel semicirculi, circa cujus centrum fit regula mobilis dioptris instructa. Ex: gr: Si requiratur angulus ACB; in puncto C ita statue centrum quadrantis, ut per dioptras regulæ superimpositæ ejus lateri CO, conspiciatur punctum B. Dein, quadrante manente in eodem situ, moveatur regula donec per dioptras appareat punctum A; & erit arcus LO, cujus gradus notantur in margine instrumenti, mensura anguli quæsiti ACB.

Nota 3.° Praxim explicatam adhiberi perinde posse, sive distantia mensuranda sit horizonti parallela, sive ad illum inclinata.

SECUNDUS MODUS.



Sit, ut priùs, BA latitudo Fluminis; BC altitudo oculi mensoris. In triangulo ABC angulus B est rectus, angulus C invenietur ope quadrantis, & latus BC vel notum est, vel facilè innotescet. Ex his tribus (*per. 3. c. 2. b*) notum fiet latus BA, distantia quæsita.

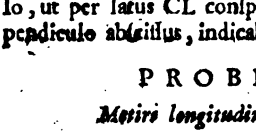
Nota 1.° Si distantia proposita BA sit magna, expedire ut mensor conscendat locum editum, tumulum vg. vel turrim, cujus nota sit altitudo.

Nota 2.° Angulum ACB [idemque est in omnibus triangulis, quorum plana sunt horizonti perpendicularia] inquiri debere quadrante ascurato, à cujus centro C dependeat perpendiculum CO. Nam si quadrantis centrum C ita admoveas oculo, ut per latus CL conspicias punctum A, arcus LO à perpendiculo abscissus, indicabit quantitatem Anguli C.

PROBLEMA II.

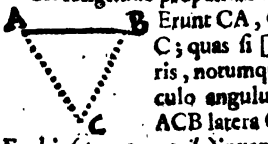
Metiri longitudinem penitùs inaccessam.

PRIMUS CASUS.



Proponatur distantia inaccessa BA jacens in directum respectu mensoris constituti in C. erit CBA linea recta, & CB, CA distantiæ accessibiles in C; quibus [*per Probl: præced:*] inventis, ex CA subtrahatur CB, eritque residuum BA distantia quæsita.

SECUNDUS CASUS.

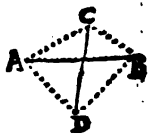


Sit longitudo proposita BA, obversa mensori existenti in C. Erunt CA, CB duæ distantiæ accessibiles in C; quas si [*per Probl: præced:*] investigaveris, notumque feceris quadrante vel semicirculo angulum C; cognoscentur in triangulo ACB latera CA, CB, & angulus contentus C. Ex his (*per. 3. c. 3. b*) invenietur longitudo quæsita BA.

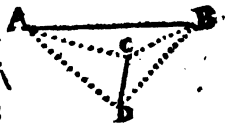
PROBLEMA III.

Duarum Urbium distantiam inquirere.

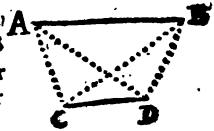
Sint duæ civitates A & B ex planitie vel intermediâ vel adjacente conspicuæ, in qua eligantur duæ stationes C & D, quarum intervallum CD mensuretur vel mechanicè, vel *per Probl.* 1.



Deinde exploratis in stratione C angulis ACD, BCD, & in stratione D angulis CDA, CDB [quod fieri debet, si distantia AB sit notabilis, quadrante magno & exquisito;] quæratur resolvendo triangulum ADC, latus AD; &, resolvendo triangulum BDC, latus BD.



Denique notis in triangulo ADB lateribus AD, BD, cum angulo comprehenso ADB [qui in 1.o & 2.o casu conflatur ex angulis CDA, CDB priùs inventis, & in 3.o casu habetur subtrahendo CDA ex CDB] invenitur latus AB distantia quæsita.



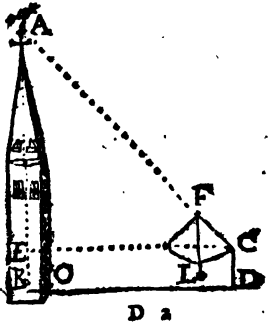
Quod dictum est de duarum Urbium distantia, eodem modo applicatur quorumvis locorum intervallo.

Altitudinum mensura.

PROBLEMA IV.

Metiri altitudinem accessibilem in Plano positam.

Sit AB altitudo mensuranda; Mensor in D, ejusque oculus in C. Ex C intelligatur duci rectæ CE parallela & æqualis lineæ horizontali DB; & erit in triangulo ACE angulus E rectus, utpote æqualis (*per.* 27. *l.* 1.) angulo B (*ex suppos.*) recto. Deinde quia lineæ FL, AB sunt horizonti perpendiculares, adeoque (*per.* 6. *l.* 11.) parallelæ, anguli A & CFL sunt (*per.* 27. *l.* 1.)



æquales ; invento igitur per quadrantem CFL, notus erit angulus A Denique latus CE, seu DB cognoscetur modo aliquo superiùs explicato. Ex his tribus reperietur (*per. 3. e. 2. h*) latus AE ; cui addatur EB seu CD altitudo oculi mensoris, & nota erit AB altitudo quæsita.

Assumpsi lineas CE, DB, item CD, ED esse æquales, quod sic ostendo. Quia CD, EB sunt perpendiculares horizonti, sunt (*per. 6 l. 11.*) parallelæ. Sed etiam CE, DB sunt (*ex suppos.*) parallelæ : ergo (*per. 35. def. l. 1.*) DBEC est parallelogrammum : ergo (*per. 34. l. 1.*) latera ejus oppositа CE, DB, & CD, EB sunt æqualia.

Nota quòd, si explorans distantiæ angulum CAE collimes in aliquod punctum A unde demissum perpendiculum AB cadit intrà ipsam turrim ; tunc distantiæ DO, mechanicè vel aliter notæ, adjicienda sit linea OB, quæ inter perpendiculum & exteriorem turris superficiem intercipitur.

PROBLEMA V.

Altitudinem accessibilem sine trianguli resolutione invenire.

Quia non semper præsto sunt tabulæ, nec vacat semper analysim Trigonometricam adhibere, subjicio alios modos faciles, quibus obviæ altitudines inquirantur.

PRIMUS MODUS. *Vide fig: prac:*

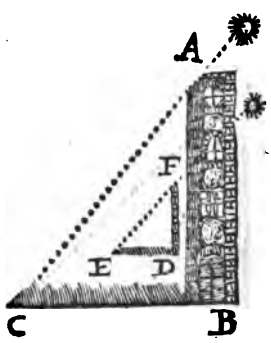
Per quadrantis dioptras collimans in Turris AB fastigium A, accede vel recede donec angulus CFL, adeoque & angulus A sit 45 gr. Tunc metire distantiam CE, eique adjice CD seu EB ; eritque summa AB altitudo quæsita.

Nam in triangulo ACE angulus E est rectus : ergo (*per 1. Corol. p. 32. l. 1.*) anguli A & C simul adæquant unum rectum. Cùm igitur A sit 45 gr. anguli A & C sunt æquales : ergo (*per 6. l. 1.*) æquantur latera AE, CE. Cætera per se patent.

SECUNDUS MODUS.

Baculum FD in æquales partes divisum erige perpendiculariter in D ; & erunt, sole lucente, duo triangula similia ABC, FDE. Nam anguli B & D sunt recti, C & E sunt

anguli ejuſdem altitudinis ſolis: ergo (*per* 2. *Corol. p.* 4. *l.* 6.) ut ED umbra baculi, eſt ad FD baculi longitudinem: ſic BC umbra turris, ad AB turris altitudinem.



Idem faciliùs obtinebis metiendo turris umbram quando ſol altus eſt 45 gr. Tunc enim turris umbra, & altitudo ſunt æquales, ut ex dictis patet. Hæc autem ſolis altitudo obſervari poteſt Quadrante vel Analemmate; vel [ſi iſta deſint] deſcribendo in plano horizontali circulum, in cujus medio erigatur ſtylus radio æqualis, planoque circuli rectus; nam ſtyli hujus umbra attinget præcisè circumferentiam circuli, quando ſol 45 gradus ſupra horizontem attollitur.

PROBLEMA VI.

Metiri altitudinem acceſſibilem, quamvis non ſit in plano.

Sit Turris AB (vide *Fig.* 2. *Probl.* 10.) in qua ſi lapidum junctura non ſint horizontales, ducatur linea horizontalis CD, quæ menſuretur mechanicè, ſicut & altitudo IC. Dein ex C obſervetur quadrante angulus CAD, ex quo & angulo recto D, & latere DC notis, invenitur (*ut ſuprà*) AD; cui addita CI vel DB, dat AB altitudinem quæſitam.

Eodem ferè modo Menſor in turris faſtigio poſitus inveniet illius altitudinem; eademque praxis inſervire poteſt ad explorandam, foſſæ vg. profunditatem.

PROBLEMA VII.

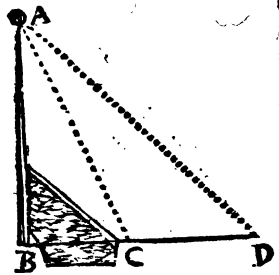
Metiri altitudinem inacceſſam.

PRIMUS MODUS.

Per *Probl.* 2. exploret Menſor ſuam ab altitudine propoſita diſtantiam; & eâ notâ, *per Probl.* 4. vel 5. ipſam altitudinem.

SECUNDUS MODUS.

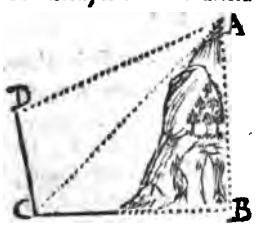
Inventio in ſtatione C angulo CAB, retrocede ad diſtantiam ſatis magnam (ut ſit inſtrumento perceptibilis differentia angulorum CAB, DAB) *vg.* ad paſſus 150, in D: & in D, notum fac angulum DAB.



Jam quia angulus B eſt rectus, patet (*ex 2. c. 2. h*) lineas CB, DB eſſe, reſpectu ſinûs totius AB, tangentes angulorum CAB, DAB. Horum igitur tangentibus quæſitis, fiat ut DC tangentium differentia, ad AB ſinum totum: ſic paſſus 150, ad aliud; & erit quartus terminus altitudo quæſita AB.

TERTIUS MODUS.

Sit alicujus Montis altitudo perpendicularis AB intra ipſius viſcera inacceſſibilis & inviſibilis. Eligantur duæ ſtationes collaterales C & D, quarum intervallum CD, ſicut & anguli DCA & D inquirantur ut ſuprà. His notis in triangulo DAC, inveniatur latus AC. Deinde in triangulo CAB, ex angulo recto B; & latere oppoſito AC, & angulo CAB vel ACB qui quadrante explorabitur, inveſtigetur AB altitudo quæſita.



Corollaria.

1. Ex iiſdem notis in triangulo CAB innoteſcet latus CB, diſtantia Menſoris ab AB perpendiculari montis altitudine.

2. Eodem modo metiris nubis altitudinem: debent tamen, propter nubis mobilitatem, eſſe duo Menſores; unus in C, alter in D, qui ſimul, dato ſigno, in nubem collimantes, obſervent angulos C & D.

Notandum.

Nisi summa adhibeatur cura, facilè erratur in angulis investigandis, unde in reliquam operationem error frequenter derivatur. Ad incommodum hoc avertendum, duo præcipuè conferent; Primum, si in instrumentorum delectu & usu magna adsit diligentia: Secundum, si assumatur stationum intervallum ita proportionatum longitudini mensurandæ, ut anguli fiant sensibiles. Hinc stationes collaterales aliis præstant, quia minori indigent intervallo, ut angulorum quantitas determinetur. Hinc etiam praxis ultima, quâ montis altitudinem investigavimus, videtur reliquis anteferenda.

PROBLEMA VIII.

Distantias & altitudines per Scalam Geometricam invenire.

Longitudines hactenus investigavimus resolvendo triangulum aliquod, cujus nota fuerunt vel duo latera cum uno angulo, vel anguli duo cum aliquo latere. Si idem triangulum, quodcunque sit, resolvatur methodo explicatâ *Cap.* 4. eadem Problematis solutio per scalam obtinebitur.

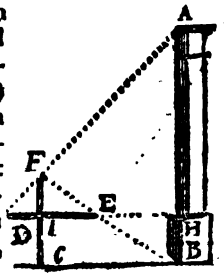
PROBLEMA IX.

Idem præstare per Crucem Geometricam.

Crux Geometrica (quæ vulgò *Baculus Jacobaus* dicitur) constat duabus Regulis DE, FC in æquales partes divisis, & ita inter se aptatis, ut (sive brachium DE moveatur sursum, sive deorsum, sive protrudatur, sive retrahatur) semper efficiant angulum rectum I.

PRIMA PARS.

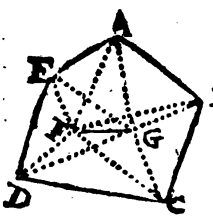
Sit mensuranda distantia CB. In C erigatur perpendiculariter (quod fit applicando normam vel quadrantem lateribus instrumenti) regula FC: alterum verò brachium DE impellatur vel retrahatur, donec per extrema F & E appareat punctum B. Sic fient duo triangula FIE, FCB habentia angulum ad F communem, & angulos FIE, FCB (*ex constr.*) rectos, adeoque



PROBLEMA XI.

Agri vel cujuſvis Figuræ rectilineæ Ichnographiam deſcribere, & dimenſiones invenire.

In agro duas tales ſtationes elige, ut recta per eas ducta in nullum agri angulum incurrat; & menſurato mechanicè, vel aliter, ſtationum intervallo, ex ſcala deſumatur recta FG continens tot ſcalæ partes, quot paſſus vel pedes continet illud intervallum.



Dein rectam FG in tabulam transfer, quam ſtatue horizontaliter in alterutra ſtatione, ita ut lineæ FG unum extremum F immineat dictæ ſtationi, & aliud extremum G in alteram ſtationem B dirigatur ope regulæ dioptris inſtructæ. Tunc regulam dirigens à puncto F ad ſingulos agri angulos, duc verſùs eos per latus regulæ, rectas FA, FB, FC, FD, FE.

Ad eundem modum collocatâ tabulâ horizontaliter in ſecunda ſtatione, ita ut G immineat huic ſtationi, & F in alteram dirigatur; ex G ducantur verſùs ſingulos agri angulos rectæ occurrentes prioribus in punctis A, B, C, D, E, quæ jungantur lineis AB, BC, CD, DE, EA; & erit figura ABCDE Ichnographia quæſita, agro ſeu plano propoſito omnino ſimilis. Unde

1.° Quot ſcalæ partes continet quævis linea in tabula, tot paſſus vel pedes continet agri latus correſpondens.

2.° Quot ſcalæ partes quadratas continet Ichnographia; tot ſunt paſſus vel pedes quadrati in agro menſurando.

3.° Anguli in tabula æquantur angulis in agro correſpondentibus; unde (*per ſchol. 23. l. 1.*) ex tabula quilibet agri angulus cognoſcetur.

Ratio horum omnium eſt, quòd triangula quæ in tabula deſcribuntur, ſint (ut patet *ex conſtr.*) ſimilia totidem triangulis in agro; adeoque omnes lineæ in tabula ſunt ad agri latera correſpondentia, ſicut FG ad ſtationum intervallum. Sed FG (*ex conſtr.*) continet præciſè tot ſcalæ partes, quot pedes vel paſſus continet ſtationum diſtantia: ergo &c.

CAPUT VI.

Inveſtigantur Diametur & Circumferentia Sphæra Terreſtris: item diſtantia Solis & Lunæ à Terra, eorumque diametri.

Legantur ex Euclidis libro 3. Prop. 18. 20. 21. 22. 31. 32. 36 cum Corollariis.

Ex libro 5. Prop. 7. 9. 11.

Ex libro 6. Prop. 14. 16. 17. cum Corollariis.

PROBLEMA I.

Terræ diametrum invenire.

Eligatur juxta mare mons excelſus, cujus altitudo perpendicularis ſit AB, quæ producta tranſit per terræ centrum Z: ſit etiam AC radius viſualis horizontem ſtringens, id eſt ſphæram terreſtrem tangens in C.

Quadrante magno & accurato exploretur angulus A; quod ut ſecuriùs fiat, captetur ortus vel occaſus alicujus ſtellæ, ſeu potiùs Solis vel Lunæ, quando extremus tantùm aſtri limbus ſupra horizontem eminet. Nam tunc collimans in aſtrum, ſimul collimabis in aliquod horizontis punctum C. Quæratur etiam (*per Probl. 7. c. 5.*) montis altitudo AB, quod fiat quàm accuratiſſimè.

Noto in triangulo ACZ angulo A, quia angulus C (*per 18. l. 3.*) eſt rectus, notus etiam erit (*per 4. Corol. p. 32. l. 1.*) angulus Z: ergo (*per idem*) nota eſt in triangulo CZB ſumma angulorum CBZ, BCZ; qui cùm, ob laterum ZB, ZC æqualitatem, ſint (*per 5. l. 1.*) æquales, utriuſque ſeorſim quantitas cognoſcetur. Subtrahe jam angulum ZCB ex recto ACA, & reſtabit angulus BCA notus. Adeoque in triangulo CBA noti ſunt anguli A, BCA, & (*per 4. Corol. p. 32. l. 1.*) angulus ABC, cum latere AB, ex quibus invenietur latus AC.

Denique in triangulo ACZ, ex latere AC & omnibus angulis deprehenditur latus CZ terræ ſemidiameter, quæ duplicata dat diametrum quæſitam.

Notanda.

1 Supponit hoc Problema quòd Terra, seu moles Terraquea, habeat figuram Sphæricam; quæ veritas variis modis solet demonstrari, 1.° Ex umbra terræ, quæ in Eclipsi Lunari semper apparet circularis. 2.° Quia ubicunque fueris, horizon visibilis circulo terminatur. 3.° Quia montium sublimiorum vertices soli à longè conspiciuntur; accedentibus verò deteguntur paulatim, idque eodem modo à quacunque tandem parte accesseris. 4.° Ex siderum motu, ortu & occasu respectu locorum diversorum, aliisque cæli phænomenis, quæ (ut patebit consideranti) quamcunque figuram non Sphæricam terræ attribueris, fierent longè aliter, quàm de facto contingunt.

2. Supposuimus hîc, quod & superiùs sæpè, centrum gravium idem esse cum centro terræ. Hoc autem est ab experientia manifestum; nam videmus gravia ubivis locorum ferri deorsum motu ad horizontem perpendiculari, & ædium parietes citò collabi, si à situ perpendiculari declinent. Hæc fusiùs explicata vide apud Ricciolum *in Almagesto. libro 2. cap. 1. §. 4.*

3. Supposuimus *cap. 5.* in altitudinum inquisitione angulum ad horizontem CBA esse rectum; & meritò, quia in distantiâ non valdè magnâ differentia est insensibilis. Est tamen reverà recto major; nam cum angulo acuto CBZ adæquat (*per 13. l. 1.*) duos rectos. Habetur autem justa mensura anguli CBA, ipsi addendo dimidium anguli Z.

4. Observari etiam debet, quòd distantia quæ invenitur per analysim trianguli plani, semper sit via brevissima, seu linea recta connectens ea loca, quorum distantia investigatur. *Ex. gr.* Si exploretur in Sphæra terrestri distantia inter C & B, non invenietur arcus CB, qui ob terræ rotunditatem est vera distantia; sed illius subtensa, seu recta CB, quæ cùm minor sit arcu, aliquantulum deficit à distantia quæsita. Quia verò defectus ille in distantiâ non valdè magnâ est insensibilis, ideò meritò negligitur. Quòd si contingat ut proponatur distantia maxima, eique requiratur par accuratio; sic facilè obtinebitur. 1.° Quæratur (*juxta hoc, vel sequens*

Probl.) angulus Z, seu arcus CB, qui sit vg. 1 gr. 30′. 2.° Positoquòd ZB Terræ semidiameter contineat qualescunque partes 10000000; resolvendo triangulum ZBC, invenietur subtensa CB esse ad latus ZB, ut 261792 ad 10000000; adeoque recta CB est ad totam diametrum ut 261792 ad 20000000. 3.° Quia diameter circuli (vide P. Tacquet *scholia in Prop. 6. selectarum ex Archimede*) est ad circumferentiam, ut 7 ad 22, vel exactiùs ut 113 ad 355, ut habeatur proportio inter diametrum sic divisam & totam circumferentiam, fiat ut 113, ad 20000000; sic 355 ad aliud, & invenietur tota circumferentia ad diametrum, ut 62831859 ad 20000000. 4.° Fiat ut 360 gr. ad 1 gr. 30′, seu ut 21600′ ad 90′, ita circumferentia 62831859 ad aliud, & quartus terminus 261799 indicabit rationem arcûs CB ad diametrum 20000000. Sed ex dictis subtensa CB est ad diametrum, ut 261792 ad 20000000: ergo subtensa CB, est ad arcum CB, ut 261792 ad 261799. 5.° Ponamus quòd subtensa CB inventa fuerit passuum 96544 & pedum $2\frac{1}{2}$, id est (supponendo pedem dividi, uti communiùs dividi solet, in 12 digitos) digitorum 5792670; Dicendo, ut subtensa CB 261792 ad digitos 5792670: sic arcus CB 261799 ad aliud; invenietur arcus CB continere digitos 5792823 seu passus 96547 $\frac{3}{60}$; adeoque differentia inter arcum & subtensam est passus 2. pedes $2\frac{3}{4}$.

Hinc confirmatur quod diximus, viz: excessum arcûs supra subtensam sine dispendio negligi in mediocri intervallo, cùm in distantia Mill. Bon: 96 $\frac{1}{2}$ seu leucarum 32 circiter, sit tantilla differentia.

Corollaria.

1. Notâ terræ semidiametro ZC seu ZB, & altitudine oculi AB, sic invenitur amplitudo horizontis Physici. *Vide definitionem 4. cap. 5. P. 2.*

In triangulo ACZ angulus C est rectus, latus CZ terræ semidiameter, & latus AZ habetur addendo oculi altitudinem

AB semidiametro terrestri ZB, & his ejusbus invenitur angulus Z, adeoque & arcus CB, qui est dimidia amplitudo horizontis Physici, & ad millaria reducetur modo statim explicando.

2. Eodem modo determinatur distantia (CB) ex qua quævis altitudo nota (AB) potest conspici: & è converso, notâ distantiâ invenitur altitudo.

Nam si detur altitudo AB, nota erunt in triangulo CAZ, latera CZ, AZ cum angulo recto C; ex quibus invenitur angulus Z, seu arcus CB distantia maxima ex qua punctum A potest conspici.

E converso, si cognoscatur distantia seu arcus CB, notus erit angulus Z, & latus CZ cum angulo C recto. Ex his tribus quæratur AZ ut subtractâ BZ terræ semidiametro, inveniatur AB altitudo quæsita.

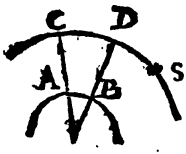
3. Ex eadem figura colligitur quòd in distantia magna non rectè exploretur angulus distantiæ CAB quadrante & perpendiculo, nisi correctio aliqua adhibeatur. Nam qui ex C collimaret in A, tendente perpendiculo ad centrum Z per rectam CZ, inveniret angulum LCZ, qui (*per* 32. *l.* 1.) æquatur angulis Z & CAB quæsito. Subtrahi igitur debet ex angulo invento LCZ angulus Z, qui (*ex dicendis*) facilè innotescet notâ distantiâ CB.

4. Ex terræ diametro notâ deducitur ejus circumferentia. Securior tamen est terrestris circumferentiæ investigatio immediata, quæ sequitur.

PROBLEMA II.

Terræ circumferentiam invenire.

PRIMUS MODUS.

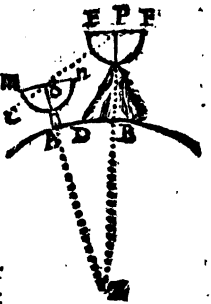


Eligantur sub eodem meridiano duo loca A & B, quorum distantia diligenter exploretur. Sit loci A, Zenith C; loci B, Zenith D. Jam, si in utroque loco observetur alicujus stellæ S altitudo meridiana, nota erunt altitudinum complementa

SD, SC; eorumque differentia CD; adeoque & arcus terrestris AB, similis arcui CD. Fiat igitur ut arcus AB, ad locorum distantiam; sic tota terræ circumferentia, seu gradus 360. ad aliud; & ostendet quartus terminus quot sint milliaria in toto terræ ambitu.

SECUNDUS MODUS.

Sit P locus in monte à longè conspicuus, & A turris in planitie milliaribus aliquot distante. Montis & turris altitudines perpendiculares PB, OA concurrentes in terræ centro Z, inter quas sit distantia horizontalis AB. His positis,



1.° Quadrante vel semicirculo ita collocato in P ut ejus latus EF sit horizontale, adeoque rectum lineæ PB, metire inclinationis angulum CPB, quem efficit radius visualis PC radens turrim AO; dein Telescopio observetur illud in turri punctum O, quod stringit radius visualis PC.

2.° In parte turris observatâ O, idem instrumentum ita fiatur, ut latus MN sit horizontale, & rectum lineæ OA. Deinde collimans in P, ubi prima facta est observatio, nota exactè elevationis angulum AOP, cujus supra rectum excessus NOP (*per.* 15. *l.* 1.) æquatur oppositò ad verticem MOC.

3.° In triangulo POZ, inventis jam angulis OPZ, POZ, notus erit (*per.* 4. *Corol. p.* 32. *l.* 1.) tertius Z: ergo notum etiam erit quot gradus vel minuta contineat arcus AB. Si igitur (*per Probl.* 3. *c.* 5.) quæratur AD, & (*per* 1 *Corol. Probl.* 7. *c.* 5.) DB, ut cognoscatur in milliaribus vel passibus tota distantia AB; invenientur (*ut in* 1. *modo*) milliaria vel passus quos continet tota terrestris Sphæræ circumferentia.

Scholium.

Ex hac operatione sæpiùs peracta, tribuit Ricciolus terræ circumferentiæ Milliaria Bononiensia 23170, & Passus 680; semidiametro terræ Mill. Bon. 3689. Passus 598: Uni Gradui

circuli maximi Mill. 64. Paſſus 363: Areæ circuli terreſtris maximi Mill: quadrata 427457247: ſuperficiei terreſtri Mill: quadrata 170980988: ſoliditati terræ Mill. cubica 21028703785.

Mathematici Pariſienſes apud Ozanam Tom. 5. pag. 149 aſſignant uni gradui Paſſus Bon: 58481, ſeu Mill. Bon. $58\frac{1}{2}$ circiter. Undè ex illorum calculo continebit terræ ambitus Mill: Bon. 21053, & Paſſus 160.

Has terræ dimenſiones menſuris Bononienſibus expreſſas ad Patriæ ſuæ milliaria quivis facilè reducat, modò determinetur illa quæ eſt inter aliquas menſuras notas, pedes *vg.* proportio. Cùm enim Milliare Bon: contineat 1000 paſſus, & paſſus ſinguli 5 pedes; *per Auream Regulam*, notâ pedum proportione, conſtabit paſſuum & milliarium proportio; modò ſciatur quot pedes vel paſſus ad alicujus regionis milliare requirantur.

Ut facilius peragatur hæc operatio, pedum celebriorum proportionem hîc ſubjicio. Quarum partium continet *Pes Bononienſis* 1686, earundem continet *Romanus antiquus* 1331. *Pariſienſis* 1440. *Londinenſis* 1350. *Leodienſis* 1320 *Rhynlandicus ſeu Leydenſis* 1390. *Antuerpienſis* 1277. *Bavarus* 1284. *Amſtelodamenſis* 1256. *Lovanienſis* 1263. *Norimbergenſis* 1353. *Venetus* 1557.

Hoc ſemel invento, quot alicujus loci milliaria correſpondeant uni gradui maximi terræ circuli, *per Regulam Auream* invenientur gradus correſpondentes cuicunque intervallo noto: & è converſo.

Ex: gr: Inquiratur arcus correſpondens milliaribus 342. Fiat ut mill: $64\frac{363}{1000}$ ſeu paſſus 64363, ad 1 gradum ſeu 60 min: ſic mill: 342 ſeu paſſus 342000, ad aliud; & quartus terminus 5 gr. 18′ erit quantitas arcûs quæſiti.

PROBLEMA III.

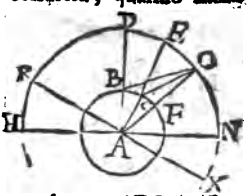
Lunæ à Terra diſtantiam metiri.

Eligantur duo loca B & C ſub eodem Meridiano HDX, in quo ſit Lunæ locus O. Loci B, Horizon ſit HN, Zenith D.

Loci

Loci. C, Horizon RX, Zenith E. His positis,

1.° Duo mensores eâdem nocte conflictâ, quando Luna appellit ad Meridianum, metiantur angulos distantiæ Lunæ à suo quisque Zenith, *viz:* angulos OBD, OCE.



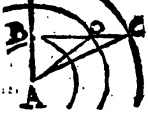
2.° Inquiratur distantia locorum B & C, ut sciatur arcus BC, seu angulus BAC; & nota erit (*per.* 4. *Corol. p.* 32. *l.* 1.) summa angulorum ABC, ACB, qui cùm (*per.* 5. *l.* 1.) sint æquales, singuli cognoscentur. Ex omnibus angulis & BA terræ semidiametro, quæratur subtensa BC.

3.° Angulis ABC, OBD jam notis, cognoscetur intermedius CBO eorum complementum ad duos rectos: & noto angulo ACB, scietur BCE ejus complementum ad duos rectos; cui si addideris OCE, notus erit BCO. Ergo in triangulo OCB ex angulis BCO, CBO, & latere BC, invenientur BO Lunæ distantia à loco B, & CO ejusdem distantia à loco C.

4.° In triangulo ABO, ex notis lateribus AB, BO, & angulo ABO conflato ex notis ABC, CBO, invenitur AO distantia Lunæ à centro terræ; & (subtractâ AF) FO, distantia Lunæ ab F, terræ puncto sibi directè subjecto.

Notandum.

Lunæ aliorumque Planetarum variatur sæpe à terrâ distantia. Hoc colligitur 1.° ex eo quòd (etiam abstrahendo à refractione) nunc sub majori, nunc sub minori angulo conspiciantur. 2.° Ex varietate Parallaxium seu angulorum AOB, ACB, quos in centro Planetæ efficiunt duo radii, quorum unus ducitur ex A centro terræ, alter ab oculo B in terræ superficie posito.



Quòd autem, dum Planeta vicinior est in O, major sit parallaxis BOA, quàm BCA parallaxis ejusdem dum remotior est in C, patet; nam (*per.* 32. *l.* 1.) angulus BOA æquatur internis BCA, OAC; adeoque excedit BCA.

Riccioloo ex observationibus plurimis relatis *libro 4. Almagesti. pag. 225.* deducit Lunæ distantias sequentes.

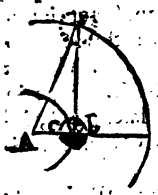
In Quadraturis Lunæ	Apogæa 66 - 42′.	Terræ	
	Perigæa 51 - 20.		
In Syzygiis Lunæ	Apogæa 64 - 15	Semidiametros.	
	Perigæa 53 - 45		

PROBLEMA IV.

Solis à Terra distantiam investigare.

Sit in Sphæra terrestri oculi locus A, Lunæ centrum *o*, Solis centrum S.

1.o Quando Luna dichotoma est, seu apparet dimidiâ sui parte illuminata, planum *b o c* separans partem lucidam ab obscura, si produceretur, transiret per oculum existentem in A; adeoque radius visualis A-*o*, transiens per Lunæ centrum, est in illo plano. Patet; quia aliter una semissis hæmisphærii illuminati non occulteret aliam.



2.o Recta S *o*, Solis & Lunæ dichotomæ centra connectens, est perpendicularis plano *b o c*; adeoque (*per. 3. def. l. 11.*) etiam radio visuali A *o*, existenti in illo plano. Patet; nam hæmisphærium illuminatum, adeoque & ipsius basis, seu planum *b o c*, soli directè obvertitur. His præmissis,

In triangulo AS*o* latus A*o* est Lunæ à terrâ distantia (*per præced.*) notâ; angulus S*o*A est rectus, & angulus *o*AS ad dichotomiæ momentum debet Instrumento exquisito explorari. Ex his tribus invenietur SA, Solis distantia à terra.

Hanc aliasque distantiæ solaris investigandæ methodos fusiùs explicatas vide apud Ricciolum *in Almagesto, l. 3. c. 7.* cum ipsius, & præcipuorum Astronomorum de Solis distantia judiciis. Tandem (*Astronomia Reformata l. 1. c. 13.*) constituit Solis distantiam maximam terræ semidiametros 7427. minimam 7173.

Scholium.

Reliquorum Planetarum & Fixarum distantiam investi-

gare operosum est, & multorum notitiam prærequirit, quorum explicatio alterius est loci. Subjiciam proinde tabulam distantiarum quas ex suis observationibus infert Riccioli (*Astron. Reform. l.* 10. *c.* 15.)

		maxima.	*minima.*	
A Terra distantia.	Mercurii	10435.	4948	Terræ Semidiametri
	Veneris	12754.	1946.	
	Martis	19576.	2670.	
	Jovis	45669.	28864.	
	Saturni	81105.	58207.	

Si Fixæ haberent parallaxim saltem 2 secundorum, inde deduceretur earum distantia à Terra major Terræ Semidiametris 100000. Cùm igitur parallaxis sit ita exigua, ut nullâ Astronomorum industriâ unquam potuerit observari, certum est earum distantiam esse multò majorem. Quanta verò sit, nequit certò determinari.

Fixarum distantia duplicata dat diametrum Firmamenti majorem Terræ semid. 200000, adeoque continentem ad minimum Passus 737919600000. Unde circumferentia Firmamenti erit Passuum plusquàm 23182445 25762. Cùm igitur Firmamentum cum omnibus stellis fixis integram revolutionem peragat 24 horis, hinc colligitur stellam sub Æquatore positam conficere minutis singulis Passus 1609892031; qui motus si conferatur cum circumferentia Terræ continente Passus 23170680, constabit stellas sub Æquatore sitas conficere minutis singulis plusquàm sexagies nonies totius Terræ ambitum. Quod tametsi mirum videri possit, est tamen infra veritatem.

PROBLEMA V.

Notâ cujuscunque sideris distantiâ, & diametro apparenti, ejus veram diametrum invenire.

Sit alicujus astri distantia AB, diameter apparens angulus CAD factus à radiis visualibus AC, AD, qui ducti ex oculi loco A sideris sphæram tangunt in C & D: sint etiam ejusdem sphæræ radii BC, BD, & diameter vera XZ.

In triangulis ABC, ABD, latera BC, BD (*per.* 18. *def. l.* 1.)

& latera AC, AD (*per.* 2. *Corol.* 36. *l.* 3. sunt æqualia, & latus AB commune: ergo (*per.* 8. *l.* 1.) anguli BAC, BAD sunt æquales, adeoque BAC est semissis anguli CAD (*ex suppos.*) noti. Ergo in triangulo ABC noti sunt anguli BAC, & BCA (*per.* 18. *l.* 3.) rectus, cum latere AB; ex quibus invenitur BC, seu BX semidiameter, quæ duplicata dat diametrum veram XZ.

Scholium.

Modus inveniendi siderum diametros apparentes, quem brevitatis causâ prætermitto, videri potest apud Dechales *Astronomia lib.* 4. *Prop.* 27. &c. Ex illis autem quas assignat Ricciolus, & distantiis suprà positis, deducuntur eorum veræ diametri ad terræ diametros se habere,

Solis. ut 33 ad 1. *Lunæ.* ut $26\frac{7}{2}$ ad 100. *Mercurii* ut 26 ad 100. *Veneris.* $1\frac{1}{10}$ ad 1. *Martis* ut 100 ad 59. *Jovis* ut 9 ad 2. *Saturni* ut $4\frac{3}{10}$ ad 1. *Sirii* $4\frac{2}{25}$ ad 1. *Capellæ.* ut $3\frac{94}{100}$ ad 1.

Quia sphærarum proportio triplicata est proportionis diametrorum, (*vide apud P. Tacquet. Corol. p.* 18. *lib.* 12. *Euclidis.*) infertur Sphæram Terrestrem esse ad Solem, ut 1 ad 35937: ad Lunam, ut 51 ad 1: ad Mercurium, ut 57 ferè ad 1: ad Venerem, ut 100 ad 133: ad Martem, ut 48 ad 10: ad Jovem, ut 8 ad 729: ad Saturnum, ut 1 ad 795: ad Sirium, ut 1 ad 83 ferè: ad Firmamentum denique, ut 1. ad 1000000000000000.

Ex dictis pronùm erit observare quàm exilis sit Terrarum orbis, si cum mundi spectabilis amplitudine conferatur. Quid porrò erit si comparetur cum cælo empyreo, fortunatâ Beatorum sede? Si hæc eo, quo par est animo, expenderit pius Lector, sordebit profectò Terra, dum cælum & cælestia contemplatur.

PARS SECUNDA.

DE TRIGONOMETRIA SPHÆRICA.

Artem hanc in quinque capita diſtribuemus. *Primò* præmittemus nonnulla ad trianguli ſphærici proprietates intelligendas neceſſaria. *Secundò* trianguli ſphærici rectanguli, *Tertiò* obliquanguli ſolvendi modum indicabimus. *Quartò* præ cæteris inquirenda laterum & angulorum ſpecies, regulas aliquas præſcribemus. *Quintò* proponemus Problemata aliquot nobiliora, quorum ſolutione præcedentium Capitum doctrina illuſtrabitur.

CAPUT I.

Præmittuntur aliqua ad Trigonometriam Sphæricam.

DEFINITIONES.

1.a *Sphæra* eſt figura ſolida, unicâ ſuperficie comprehenſa, ad quam omnes rectæ ductæ ab aliquo intra ſphæram puncto (quod *Centrum* dicitur) ſunt æquales.

2.a *Circulus maximus* eſt quò major non poteſt in ſuperficie ſphæræ deſcribi.

3.a *Polus* circuli eſt punctum in ſuperficie ſphæræ, ab iſtius circuli circumferentia diſtans undique æqualiter.

4.a *Angulus Sphæricus* (de quo hîc agimus) fit à duobus circulorum maximorum arcubus; & tot continet gradus, quot continet angulus quem faciunt circulorum plana.

Præter pleraque in Parte præcedenti recenſita, ex Euclide ad Trigonometriam Sphæricam requiruntur ex Libro 11 Prop. 20. 47. cum 4. & 5. Coral. Ex Libro 3. Prop. 7. 9. 10.

13. 29. cum 1. Coról. Ex Libro 5. Prop. 22. 23. Ex Libro 11. Prop. 9. 15.

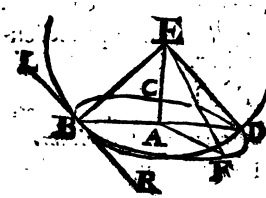
Nota. Quòd tametsi quæ capite hoc continentur ad trianguli sphærici proprietates penitiùs inspiciendas multùm juvent, non tamen omnia ad trianguli resolutionem intelligendam sint absolutè necessaria. Tyroni proinde suaserim, ut Definitionibus & prioribus 7 Propositionibus contentus, Caput secundum auspicetur; ad reliqua, ubi posteà citantur, commodiùs recurret.

PROPOSITIO I.

Si sphæra plano quomodocunque secetur, sectio communis est circulus.

Demonstr. 1.° Si planum transeat per centrum, patet, nam omnes lineæ à centro sphæræ ad superficiem ductæ sunt æquales.

2.° Si non transeat per centrum, ad planum illud à centro ducatur perpendicularis EA, & ex A ducantur ad sectionis perimetrum BCDF quotcunque rectæ AB, AF, AD, quibus occurrant radii sphæræ EB, EF, ED. In triangulis EAB, EAF, EAD, anguli ad A (*per* 3. *def. l.* 11.) sunt recti, & hypotenusæ EB, EF, ED (*per* 1. *def. h*) æquales: latus verò EA est commune: ergo (*per* 4. *Corol. p.* 47. *l.* 1.) latera AB, AF, AD (idemque est de omnibus quæ duci possunt ab A ad peripheriam sectionis) sunt æqualia. Ergo (*per* 18. *def. l.* 1.) sectio communis est circulus, cujus circumferentia est BCDF, & centrum A.



Corollaria.

1. Circulus maximus est qui transit per centrum sphæræ. Nam radii talis circuli sunt, ut patet, iidem cum radiis sphæræ, & centrum idem: sed radii cujuscunque circuli non transeuntis per centrum sphæræ sunt minores radio sphæræ; ut patet in circulo BCDF, cujus radii sunt (*per* 18. *l.* 1.) minores radiis sphæræ oppositis angulo recto A.

2. Circulus duci potest per quæcunque tria puncta in superficie sphæræ. Nam per illa puncta duci potest planum, cujus cum sphærâ sectio communis est circulus.

3. Circuli per illa tria puncta descripti circumferentia tota est in superficie sphæræ. Nam planum per illa tria puncta ductum facit circulum, cujus circumferentia est tota in sphæræ superficie, ut patet. Sed per eadem tria puncta non potest duci nisi unus circulus; aliter enim duo circuli se secarent aut tangerent in tribus punctis, quod est (*per* 10. & 13. *l.* 3.) impossibile: ergo.

4. Recta LR tangens aliquem in sphæra circulum BCDF, tangit ipsam sphæram: nam cùm circuli BCDF circumferentia sit tota in sphæræ superficie, & LR in eodem plano cum circulo BCDF, patet quòd LR non possit sphæram secare, quin secet circulum BCDF; adeoque non sit illius tangens.

PROPOSITIO II.

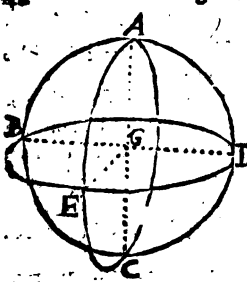
Arcus insistens arcui facit angulos duobus rectis pares, & arcus se intersecantes faciunt angulos ad verticem oppositos æquales.

Demonstr. Plana (*per 6. def. l.* 11.) faciunt eosdem angulos quos lineæ ductæ perpendiculariter ad sectionem communem. Sed lineæ istæ, quando planum insistit plano, adeoque linea lineæ, faciunt (*per* 13. *l.* 1.) angulos æquales duobus rectis: & quando plana, adeoque lineæ in planis ductæ, se intersecant, faciunt (*per* 15. *l.* 1.) angulos æquales ad verticem oppositos: ergo plana faciunt dictos angulos. Sed (*per 4. def. h*) arcus faciunt eosdem angulos, quos circulorum plana: ergo.

PROPOSITIO III.

Mensura cujusvis anguli spherici (BAE) *est circuli maximi arcus* (BE) *interceptus inter* AB, AE) *latera istius anguli, & utrinque distans* 90 *gradibus ab* (A) *anguli vertice.*

Demonstr. Quia AB, AE sunt arcus 90 graduum, anguli



AGB, AGE sunt recti : ergo lineæ EG, BG sunt perdendiculares ad planorum sectionem AC: ergo (*per def. 6. l.* 11.) angulus BGE est angulus inclinationis planorum ; adeoque (*per 4. def h*) æquatur angulo sphærico BAE : sed arcus BE est mensura anguli BGE : ergo & anguli BAE.

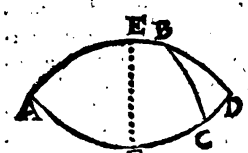
PROPOSITIO IV.

In Sphæra circuli maximi se mutuò bisecant.

Demonstr. Centrum sphæræ (*per* 1. *Corol.* 1. *h*) est centrum circulorum maximorum : ergo omnes circuli maximi habent idem centrum : ergo communis illorum sectio transit per centrum commune ; adeoque est (*per* 20. *def. l.* 1.) communis eorum diameter, eosque bisecat in duos semicirculos.

Corollaria.

1. Si trianguli sphærici ABC duo quævis latera AB, AC, quæ in uno sectionis extremo faciunt angulum A, producantur donec fiant semicirculi, concurrant in altero extremo D.

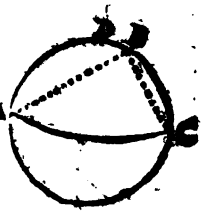


2. Dum trianguli ABC latera sic producta concurrunt, fit novum triangulum, in quo 1.o habetur angulus D æqualis angulo A, nam idem arcus EF est (*per 3. h*) utriusque mensura. 2.o Latus BC est utrique triangulo commune. 3.o Reliqua latera BD, CD sunt laterum AB, AC complementa ad semicirculum. 4.o Anguli CBD, BCD sunt angulorum ABC, ACB complementa ad duos rectos. Unde 5.o Hoc ipso quòd cognoscantur aliqui anguli vel latera in uno triangulo, cognoscuntur correspondentes anguli & latera in alio. 6.o Iidem sunt sinus correspondentium tum laterum, tum angulorum in utroque triangulo.

PRO-

PROPOSITIO V.

Polus circuli maximi distat undique 90 gradibus ab ejus circumferentia.



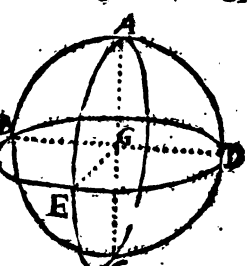
Demonstr. Si neges: Sit circuli AC Polus B, distans minùs quadrante à C. Quia ADBC est (*per 4. h.*) semicirculus, arcus ADB major est arcu BC, adeoque subtensa AB major subtensâ BC: ergo B minùs distat à C quàm ab A: ergo (*per 3. def. h.*) B non est polus.

Corollarium.

Eodem modo ostenditur circulum quemvis unicum habere polum in eodem hæmisphærio. Si possint esse duo, sint D & B. Per D & B ducatur arcus ADBC. Patet puncta D & B non posse ambo distare 90 gradibus ab A & C.

PROPOSITIO VI.

Circulus maximus (ABCD) *transiens per* (A) *polum alterius circuli maximi* (BED) *illi insistit ad angulos rectos: & insistens ad angulos rectos transit per polum.*



Demonstr. I. Pars. Sumatur arcus BE 90 graduum, & ex E ducatur arcus EA. In triangulo ABE, arcus BE (*ex suppos.*) est quadrans, & BA (*per 5. h.*) est etiam quadrans: ergo (*per 3. h.*) arcus EA est mensura anguli ABE. Sed EA (*per 5. h.*) est quadrans: ergo (*per 3. h.*) angulus ABE est rectus: ergo (*per 4. def. h.*) planum AB est rectum plano BE.

2. *Pars.* Si insistens ad angulos rectos non transeat per polum, inclinatus in unam vel aliam partem transibit per polum: ergo (*per I. partem*) sic inclinatus insistet ad angulos rectos, quod est absurdum.

PROPOSITIO VII.

Si trianguli rectanguli ſphærici (ABC) *cujus ſinguli arcus ſunt quadrante minores, latera omnia producantur donec fiunt quadrantes* (ACZ, ABX, CBP) *& jungantur puncta* P & Z *arcu circuli maximi* (PZ); *hic arcus* (PZ) 1° *erit quadrans*; 2° *tranſibit per* (X) *extremum quadrantis interjacentis* (ABX) 3° *faciet* (ad X) *angulos rectos.*

Demonſtr. 1. Pars. Quia angulus aliquis C eſt (*ex ſuppoſ.*) rectus, arcus CBP (*per 6. h.*) tranſit per polum circuli ACZ : cùm igitur CBP ſit (*ex conſtr.*) quadrans, punctum P (*per 5. h.*) eſt ille polus : ergo (*per eandem*) arcus ZP eſt quadrans.

2. *Pars.* Quia ZP tranſit per P polum circuli ACZ, angulus Z (*per 6. h.*) eſt rectus : ergo (*per eandem*) ACZ tranſit per polum circuli ZP; & quia ACZ (*ex conſtr.*) eſt quadrans, punctum A (*per 5. h.*) eſt ille polus : ergo cum ABX ſit (*ex conſtr.*) quadrans, præcisè pertingit (*per 5. h.*) à polo A ad arcum ZP, qui proinde tranſit per X.

3. *Pars.* Quia arcus XBA tranſit per A polum arcûs ZP, illi inſiſtit (*per 6. h.*) ad angulos rectos; adeoque anguli ad X ſunt recti.

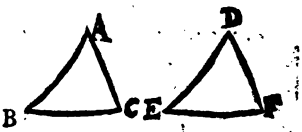
Corollarium.

Dum ſic producuntur latera, fit novum triangulum BPX, cujus latus BX eſt complementum arcûs AB; & latus BP complementum arcûs CB; & latus PX complementum arcûs XZ, id eſt (*per. 3. h.*) anguli A. Angulus verò PXB eſt rectus; angulus PBX æqualis (*per. 2. h.*) oppoſito ad verticem ABC; & angulus BPX (*per. 3. h.*) menſuratur ab arcu CZ, qui eſt complementum arcûs AC. Unde hoc ipſo quòd noti ſint aliqui anguli vel latera trianguli ABC, cognoſcuntur correſpondentes anguli & latera in triangulo BPX: & è contrà.

PROPOSITIO VIII.

Si in duobus triangulis (ABC, DEF) *æqualia ſint duo*

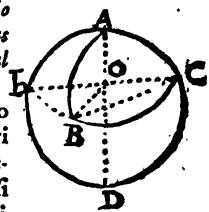
latera (AB, DE, & AC, DF) *& anguli contenti* (A & D) *æquales; vel si basis* (BC) *æquetur basi* (EF,) *& anguli adjacentes basi* (B & E, C & F) *sint æquales; tota triangula erunt æqualia.*



Hæc propositio correspondet quartæ l. 1. ejusque scholio, & eodem modo demonstratur; nam si unum triangulum alteri imponas, perfectè congruent.

PROPOSITIO IX.

Cujusvis trianguli sphærici (ABC) *duo quævis latera* (AB, AC) *sunt simul sumpta majora tertio* (BC) *item quodvis latus* (BC) *est minus semicirculo, & omnia tria latera simul sumpta minora circulo.*



Demonstr. I. Pars. Perfecto circulo AC, acceptoque arcu A*b* æquali lateri AB, ducantur chordæ BC, & *b*OC secans diametrum AD alicubi in O. Jam si semicirculus A*b* D concipiatur moveri circa communem diametrum DA donec coincidat cum plano arcûs AB, quia A*b* AB sunt (*ex constr.*) æquales arcus æqualium circulorum, punctum *b* incidet in B, & recta *b* O congruet rectæ BO. Sed (*per* 20. *l.* 1.) BO, OC, id est subtensa *b* C major est subtensâ BC: ergo (*per.* 26. *l.* 3.) arcus *b* AC seu latera AB, AC, excedunt arcum seu latus BC.

2. *Pars.* Quia BC, AC tantùm concurrunt in uno extremo C, si producantur, (*per* 1. *Corol.* 4. *h*) concurrent in altero sectionis extremo D, & erit CBD semicirculus: ergo arcus BC est minor semicirculo.

3. *Pars* DB, DA sunt (*per I. partem*) simul majora quàm AB. Sed CA, CB cum DA, DB, (*per* 4. *h*) simul æquantur circulo: ergo CA, CB cum AB sunt minora circulo.

PROPOSITIO X.

Cujusvis trianguli sphærici (ABC) *quivis angulus* (C) *minor est duobus rectis: omnes anguli simul sumpti sunt minores sex rectis, & majores duobus rectis.*

Demonstr. 1. Pars. Producatur latus BC. Anguli X & C (*per 2. b*) æquantur duobus rectis: ergo angulus C est minor duobus rectis.

2. *Pars* patet ex prima.

3. *Pars* 1° Si anguli X & B sint æquales, ecut X cum C (*per 2. b*) ita B cum C, faciet duos rectos: ergo anguli B, C, A, sunt majores duobus rectis.

2.° Si X sit minor quàm B, quia X & C faciunt duos rectos; B & C, adeoque à fortiori B, A & C, erunt majores duobus rectis.

3.° Sit X major quàm B. Tres angulos A, B, C excedere duos rectos postea commodius demonstrabimus. *Vide Coroll. Prop. 13. quæ ab hac non dependet.*

PROPOSITIO XI.

Si in triangulo sphærico (ABC) *duo latera* (AB, AC) *sint æqualia, anguli iis oppositi* (B & C) *sunt æquales: & è converso.*

Demonstr. 1. Pars. Ex AB, AC aufer arcus æquales AG, AH, & describe arcus GC, BH. Trianguli GAC, HAB habent latera AG, AH (*ex constr.*) & latera AB, AC (*ex suppos.*) æqualia, & angulum A communem: ergo (*per 8. b*) æquantur bases GC, HB, & anguli AGC, AHB; adeoque & horum complementa BGC, BHC sunt æqualia. Sed etiam latera GB, HC & GC, HB sunt æqualia: ergo anguli GBC, HCB, id est B & C sunt (*per 8. b*) æquales

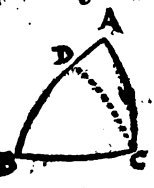
2. *Pars.* Si latera AB, AC non sint æqualia, sit AB majus. Ex AB aufer BD æquale AC, & duc arcum DC.

In triangulis BDC, BAC latus BC eſt commune, latus BD (*ex conſtr.*) æquatur lateri AC, & angulus contentus B (*ex ſuppoſ.*) angulo contento C: ergo (*per* 8. *h*) triangula BDC, BAC ſunt æqualia, pars & totum.

PROPOSITIO XII.

In triangulo quovis ſpharico (ABC) *majori angulo* (C) *opponitur majus latus* (AB:) *& è converſo.*

Demonſt. I. Pars. Fiat angulus DCB æqualis angulo B: & erunt (*per* 11. *h*) CD, BD æqualia. Sed (*per* 9. *h*) CD, DA ſimul ſunt majora quàm AC: ergo DB cum DA, ſeu AB majus eſt quàm AC.

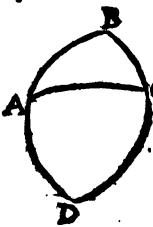


2. *Pars.* Si anguli B & C eſſent æquales, latera AB, AC eſſent æqualia (*per* 11. *h.*) Et ſi C eſſet minor quàm B, arcus AB (*per I. partem*) foret minor quàm AC: ergo ſi AB ſit major quàm AC, angulus C eſt major quàm B.

PROPOSITIO XIII.

Si trianguli ſpharici (ABC) *duo quævis latera* (AB, AC) *ſimul ſumpta ſint æqualia ſemicirculo, anguli* (B & C) *ad baſim erunt ſimul æquales duobus rectis; ſi majora ſint, anguli ſunt majores; ſi minora, anguli ſunt minores duobus rectis: & è converſo.*

Demonſtr. I. Pars. Producantur BA, BC, donec concurrant in D. Jam quia BA, AC [*ex ſuppoſ.*] & BA, AD (*per* 4. *h*) æquantur ſemicirculo; AC, AD ſunt æqualia: ergo (*per* 11. *h*) anguius ACD æquatur angulo D, id eſt (*per* 2. *Corol. p.* 4. *h*) angulo B. Sed angulus ACB cum ACD (*per* 2. *h*) æquat duos rectos: ergo ACB cum B facit duos rectos.



E converſo. Si B, id eſt (*per* 2. *Corol. p.* 4. *h*) D cum ACB æquat duos rectos, quia ACD cum eo-

dem ACB (*per* 2. *h*) idem facit; anguli D & ACD sunt æquales: ergo (*per* 11. *h*) æqualia sunt latera AC, AD. Sed AB, AD sunt (*per* 2. *Corol. p.* 4. *h*) semicirculus: ergo AB, AC æquantur semicirculo.

2. *Pars.* Si AB, AC sint semicirculo majora, AC est (ut patet ex dictis) majus quàm AD: ergo angulus D, adeoque & B, est (*per* 12. *h*) major angulo ACD. Sed ACD cum ACB (*per* 2. *h*) facit duos rectos: ergo B cum ACB facit plusquàm duos rectos.

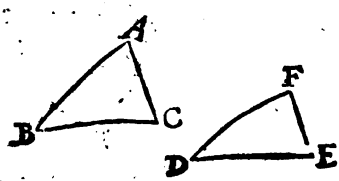
Conversa hujus, sicut & tertia pars cum suâ conversa simili ratiocinio demonstratur.

Corollarium.

Si angulus externus ACD *sit major interno opposito* B, *anguli interni* A, B, C *sunt majores duobus rectis.*

Fiat angulus ECD æqualis angulo B, & producatur BA donec concurrat cum CE. Jam angulus ECB cum ECD, id est (*ex constr.*) cum B adæquat duos rectos: ergo (*per* 13. *h*) latera BE, CE æquantur semicirculo: ergo latera AE, EC sunt simul semicirculo minora: ergo (*per* 13. *h*) anguli EAC, ECA sunt minores duobus rectis: ergo BAC, qui cum EAC (*per* 2. *h*) æquat duos rectos, est major quàm ECA. Si ergo angulo BAC addas angulum B, & angulo ACE angulum ECD æqualem ipsi B, & utrinque adjicias angulum ACB: erunt tres anguli BAC, ACB, CBA majores tribus ACE, ACB, ECD, id est (*per* 2. *h*) duobus rectis.

PROPOSITIO XIV.

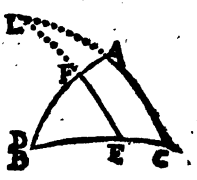


Si duo triangula sphærica (ABC, DEF) *habeant latera* (AB, AC, BC) *singula æqualia singulis* (FD, FE, DE) *anguli æqualibus lateribus oppositi sunt æquales: & è converso.*

Hujus Propositionis prima pars correspondet octavæ l. 1. Euclidis; & quemadmodum illa *per* 4. & 5. *l.* 1. *Euclidis* demonstrata est, ita & hæc eodem modo potest demonstrari *per* 8. & 11. *h.*

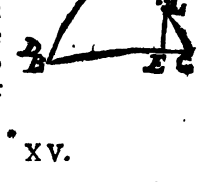
Demonstr. 2. *Pars.* Quòd triangula sibi mutuò æquiangula, sint etiam æquilatera si habeant latus aliquod commune vel æquale, patet *ex* 8. *h.* Donec igitur demonstretur contrarium, fingamus omnia latera esse inæqualia. Et

1.o Posito angulo D super æqualem B, latera DE, DF minora sint lateribus BC, BA. Producantur EF, CA donec concurrunt in puncto L. Anguli FED, FEC (*per* 2. *h.*) æquantur duobus rectis. Sed FED & ACB (*ex suppos.*) sunt æquales: ergo FEC, ACB æquantur duobus rectis: ergo trianguli LEC latera LE, LC (*per* 13. *h.*) æquantur semicirculo. Similiter anguli EFA, EFD (*per* 2. *h.*) æquant duos rectos. Sed EFD, CAB (*ex suppos.*) sunt æquales: ergo EFA, CAB æquantur duobus rectis: ergo (*per* 2. *h.*) eorum complementa LFA, LAF æquantur duobus rectis: ergo (*per* 13. *h.*) latera LF, LA æquantur semicirculo, quod est absurdum, quia arcus LC, LE jam ostensi sunt semicirculo æquales.



2.o Si latera DE, DF ponantur singula majora lateribus BC, BA; eadem fiet demonstratio.

3.o Si unum DF ponatur majus quàm BA, & aliud DE minus quàm BC; ostendetur, ut priùs, arcus LE, LC, item arcus LA, LF esse æquales semicirculo: ergo latera AC, FE vel sunt semicirculi, vel aliquod eorum majus semicirculo; utrumque verò est (*per* 9. *h.*) impossibile: ergo latera singula trianguli DEF æquantur singulis trianguli BCA.



PROPOSITIO XV.

Si in triangulo sphærico Isosceli (ADC) *ducatur chorda basis* (CD) & *per* (A) *verticem anguli oppositi transeat*

recta (MA) *chorda* (CD) *parallela, illa rectæ* (MA) *tanget sphæram in qua est triangulum* (ACD) *ita ut in eam non incidat.*

Demonstr. Per puncta A, D, C, describatur circulus & ducantur chordæ arcuum AC, AD, quæ (*per 26. l. 3.*) sunt

æquales. Jam si MA tangat circulum ACD, (*per 4. Corol. p. 1. h*) tangit sphæram: si verò neges MA esse tangentem circuli ACD, ducatur alia quævis IA quæ circulum ACD tangat in A: ergo [*per 32. l. 3.*] angulus IAD æquatur angulo C, id est [*per 5. l. 1.*] angulo D: ergo [*per 28. l. 1.*] IA & DC sunt parallelæ: sed MA, DC [*ex constr.*] sunt parallelæ: ergo [*per 30. l. 1.*] IA, MA sunt parallelæ, quod est absurdum, nam concurrunt in A. Ergo MA, & non IA, tangit circulum ADC.

LEMMA.

Si recta RG à centro ducta bisecet arcum BGC, bisecat ejus chordam BC, estque ad eam perpendicularis.

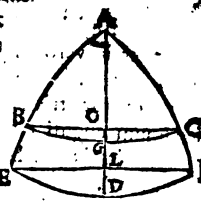
Demonstr. Quia arcus BG, GC sunt æquales, æquales sunt [*per 23. l. 1.*] anguli BRG, CRG. Æquantur etiam latera RB, RC, & latus RO est commune: ergo [*per 4. l. 1.*] bases BO, CO sunt æquales, [adeoque bisecatur BC, quod erat primum] sicut & anguli ad O, qui proinde (*per 13. l. 1.*) sunt recti: ergo CO est perpendicularis ad RO, quod erat secundum.

PROPOSITIO XVI.

Si ex arcubus æqualibus [AE, AF] *auferantur arcus æquales* [AB, AC] *& per arcuum extremitates* [B & C, E & F] *ducantur circulorum maximorum arcus* [BGC, EDF] *Planum circuli maximi* [AD] *bisecantis angulum verticalem* [A] *bisecat arcus illi subtensos* [BGC, EDF] *& horum chordas* [BGC, ELF] *quæ etiam sunt parallelæ.*

Demonstr. I. Pars. In triangulis BAG, CAG, anguli ad A

ad A [*ex suppos.*] sunt æquales, sicut & latera AB, AC [*ex constr.*] latus verò AG est commune: ergo [*per 8. b*] bases BG, CG sunt æquales. Idem similiter ostendam de ED, FD: ergo bisecantur arcus BGC, EDF.



2. *Pars.* Linea à centro ducta & bisecans arcum BC in G, bisecat [*per Lemma præced.*] ejus chordam in O. Sed hæc linea est in plano circuli AD, quod [*ex suppos.*] transit per G, & (*per 1. Corol. 1. b*) per centrum sphæræ, id est [*per idem Corol.*] per centrum circuli BGC: ergo planum circuli AD bisecat chordam BOC. Eodem modo idem demonstratur de chorda ELF.

3. *Pars.* Sequitur ex dictis in prima parte angulos ad G esse æquales, adeoque (*per 2. b.*) rectos: ergo planum circuli GC [*per 4. def. b*] rectum est plano circuli AGD. Deinde [*per Lemma præced.*] CO est perpendicularis lineæ ductæ à G ad centrum. Sed hæc linea est (*ut ostendimus in 4. b*) communis sectio circulorum AGD, BGC: ergo CO est perpendicularis sectioni communi planorum AG, CG; adeoque (*per 4. def. l. 11.*) est recta plano circuli AGD. Eodem ratiocinio ostendam FL esse rectam eidem plano AGD: ergo (*per 6. l. 11.*) lineæ CO, FL, adeoque & chordæ BOC, ELF sunt parallelæ.

CAPUT II.

De Resolutione Trianguli Sphærici Rectanguli.

PROPOSITIO I.

In omni triangulo sphærico rectangulo

Ut sinus cujusvis lateris, ad sinum anguli sibi oppositi:
Sic sinus alterius lateris, ad sinum anguli sibi oppositi.

Et è contrà.

Ut sinus cujusvis anguli, ad sinum lateris sibi oppositi:
Sic sinus alterius anguli, ad sinum lateris sibi oppositi.

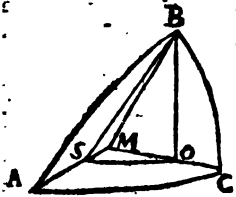
Hæc est prima regula pro resolutione trianguli sphærici rectanguli, & vocatur *Regula Sinuum seu Oppositionis.*

Nota. Quando sermo est de triangulo rectangulo, nomine

Basis intelligi hypotenusam seu arcum oppositum angulo recto : arcus verò angulum rectum continentes simpliciter vocari *Latera*.

Proponatur triangulum sphæricum rectangulum ABC : dico sinum anguli recti C esse ad sinum basεos AB ; ut sinus cujusvis alterius ad basim anguli BAC, est ad sinum lateris oppositi BC.

Sit centrum sphæræ M, & sint MA, MB, MC communes sectiones circulorum AB, BC, AC. Ab angulo B ducantur BS, BO perpendiculares ad AM, CM ; & jungantur O & S rectâ OS. Ex hac constructione habetur



1.° Quòd BS (*per def. sinûs*) sit sinus arcûs BA, & BO sinus arcûs BC.

2.° Quia angulus C est rectus, planum MBC est (*per* 4. *def. c.* 1. *h*) rectum plano MCA : ergo (*per* 4. *def. l.* 11.) linea BO, quæ est sinus arcûs BC, adoque perpendicularis diametro MC, est recta plano MCA : ergo (*per* 3. *def. l.* 11.) angulus BOS est rectus.

3.° Quia BO est recta plano MCA, planum trianguli SBO ductum per BO est (*per* 18. *l.* 11.) rectum ad planum MCA : ergo cùm MS ponatur recta ad SB, est (*per* 4. *def. l.* 11.) recta ad planum trianguli SBO : ergo (*per* 3. *def. l.* 11) MS est etiam recta ad lineam SO : ergo (*per* 6. *def. l.* 11.) BSO est angulus quem faciunt plana circulorum BA, AC : ergo (*per* 4. *def. c.* 1. *h*) angulus BSO æqualis est angulo sphærico A : sicut & angulus rectus BOS angulo sphærico C. His præmissis

Demonstr. propositio. In triangulo plano BOS, recta BS (*per* 1. *c.* 2. *P.* 1.) est ad sinum totum, seu sinum anguli oppositi BOS, ut BO ad sinum anguli oppositi BSO. Sed BS, BO sunt sinus laterum BA, BC : & anguli BOS, BSO sunt æquales angulis sphæricis C & A : ergo BS sinus lateris BA, est ad sinum totum, seu sinum anguli oppositi C ; ut BO sinus alterius lateris BC, ad sinum oppositi anguli A : & è contrà.

Quod ostensum est de angulo acuto BCA facilè demonstratur de angulo obtuso BAC ; nam productis lateribus

BC, BA, sit novum triangulum ACD, cujus anguli & latera (*per.* 2. *Corol.* 4. *c.* 1. *h*) sunt priorum complementa; & eosdem habent sinus. Cùm igitur sit in triangulo CAD, ut sinus lateris AC ad sinum anguli recti D; ita sinus lateris CD ad sinum anguli acuti CAD: erit etiam (propter sinuum identitatem) in triangulo BAC, ut sinus AC ad sinum anguli recti B: ita sinus lateris BC ad sinum anguli obtusi BAC.

Meliùs intelligetur tota hæc, uti & sequens, demonstratio, si ex charta, vel alia materia, circulorum segmenta confeceris, quæ ad angulos præscriptos erecta, circulorum in sphæra situm meliùs repræsentent.

PROPOSITIO II.

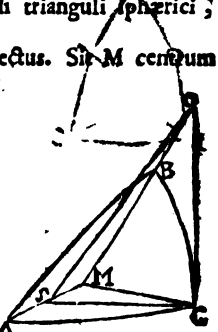
In omni triangulo sphærico rectangulo
Ut sinus totus, ad sinum cujusvis lateris circa angulum rectum;
Sic tangens anguli acuti prædicto lateri adjacentis,
Ad tangentem lateris sibi oppositi.

Et è contrà.

Ut sinus cujusvis lateris circa angulum rectum;
Ad sinum totum:
Sic tangens alterius lateris circa angulum rectum,
Ad tangentem anguli sibi oppositi.

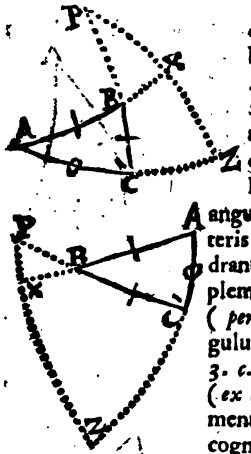
Hæc est secunda regula pro analysi trianguli sphærici; & vocatur *Regula Tangentium.*

In triangulo ABC sit angulus C rectus. Sit M centrum sphæræ, & MA, MB, MC sectiones communes circulorum AB, AC, BC. Ab angulo C ducatur CS perpendicularis ad diametrum MA, & super MC erigatur perpendicularis CO tangens circulum CB in C, & concurrens in O cum MB productâ. Demum jungantur puncta O & S.



Dico CS, sinum lateris CA circa angulum rectum, esse ad sinum totum, ut CO tangens alterius lateris

lateris noti; tertius denique tangens anguli oppositi lateri quæsito. Si verò quæratur angulus, primus erit sinus lateris adjacentis angulo quæsito; secundus erit sinus totus; tertius tangens lateris oppositi angulo quæsito.



3.o Si ita disponantur termini dati, ut neutra regula locum habeat, sicut contingit in triangulo ABC, producantur (*ut in* 1. *vel* 2. *Fig.*) omnia trianguli dati latera, donec fiant quadrantes, & ducatur circuli maximi arcus PZ. In triangulo PBX (*per.* 7. *c.* 1. *h*) angulus X est rectus; latus BP est lateris noti BC complementum ad quadrantem; & latus BX est simile complementum lateris noti AB. Ex his (*per Regulam sinuum*) invenitur angulus P, adeoque arcus CZ, qui (*per.* 3. *c.* 1. *h*) est mensura anguli P, & (*ex constr.*) lateris quæsiti AC complementum ad quadrantem; quo noto, cognoscetur latus AC.

Ad eundem modum, quandocunque nec per *Regulam sinuum*, nec *tangentium* apparet solutio; si rectè producantur latera, dabitur in novo triangulo casus dato correspondens, semper per alterutram regulam resolubilis. Dixi, *si rectè producantur*, quia dubitari sæpe potest versùs quam partem fieri debeat hæc laterum productio; ad hoc autem determinandum juvabunt regulæ sequentes.

REGULÆ

Indicantes versùs quam partem cujusvis trianguli sphærici rectanguli arcus debeant produci.

1.a Si neuter acutus angulus notetur (id est, si neuter sit vel datus vel quæsitus) per quem libuerit angulum prodaci possunt arcus.

2.a Si unus tantùm angulus acutus notetur, produci debent arcus per angulum non notatum.

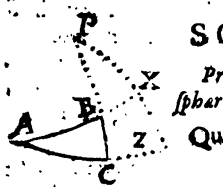
3.a Si uterque angulus acutus fuerit notatus, protrahantur arcus per angulum adjacentem alicui arcui notato.

Regulas has, & alia hactenus explicata meliùs intelliget Tyro ex Schemate ſubjecto, ubi exprimuntur caſus omnes qui poſſunt in rectangulis contingere, & modus eorum quemvis unicâ operatione reſolvendi.

Nota 1.o literam Q in ſchemate ſignificare *quæritur*. Et quamvis ſemper deſignetur aliquod latus, vel aliquis angulus quæſitus, eadem tamen ſolutio eſt alteri lateri vel angulo applicabilis.

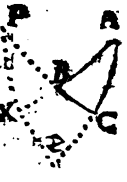
Nota 2.o quòd ſi in triangulo rectangulo, ſint quævis duo latera majora quadrante. (Hoc autem ſemper contingit, ut oſtendam *cap.* 4, quando angulus aliquis eſt obtuſus, vel latus aliquod quadrante majus) producì debeant latera majora donec fiant ſemicirculi, ſic loco trianguli propoſiti habebitur novum triangulum reſolvendum, cujus ſingula latera erunt quadrante minora, qualia requiruntur, ut per regulas datas perfici poſſit operatio.

Nota 3.o ut vitetur error, qui facilè poteſt contingere in accipiendis arcubus vel angulis pro eorum complementis ad ſemicirculum vel duos rectos; expedire ut, Tyrones præſertim, in charta vel tabula delineent triangula reſolvenda, eorumque latera producant ex præſcripto regularum; hinc enim fiet ut totius reſolutionis ſchemate oculis ſubjecto, faciliùs poſſint obſervare, quomodò ex inventis innoteſcant quæſita.



SCHEMA I.

Pro analysi triangulorum Sphæricorum rectangulorum.



Quæritur latus aliquod *a c* vel *b c*.

1	*Ex ang. oppos.* b, *& basi* ab, ♐ a c.	*Ut* sinus totus, *ad* sinum *baseos* a b. *Sic* sinus *ang. dati* b, *ad* sinum *lateris* a c.
2	*Ex ang. adjac* a, *& basi* a b, ♐ a e.	*Ut sin. compl. ang. dati* a, *ad sin. totum*: *Sic tang. compl. baseos* ab, *ad tang. compl. lat.* a c.
3	*Ex altero lat.* b c, *& basi* a b, ♐ a c	*Ut sin compl. lat dati* b c, *ad sin. totum*: *Sic sin compl basi* a b, *ad sin. compl. lat.* a c.
4	*Ex ang. oppos.* b, *& altero lat.* b c, ♐ a c.	*Ut sin. tot. ad sinum lat dati* b c. *Sic tang. ang. dati* b *ad tang. lat.* a c.
5	*Ex ang adjac.* a, *& altero lat.* b c, ♐ a c.	*Ut tang. ang. dati* a, *ad tang. lat. dati* b c: *Sic sin. tot. ad sinum lat.* a c.
6	*Ex omnibus angulis.* ♐ b c.	*Ut sin. ang. adjac* b, *ad sin. compl. ang. oppos.* a: *Sic sin tot. ad sin. compl. lat.* b c.

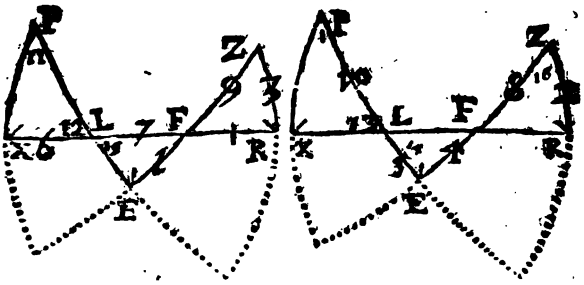
Quæritur Basis *a b*.

7	*Ex quovis latere* b c, *& ang. ei oppos.* a.	*Ut sin. ang. dati* a, *ad sin. lat. dati* b c: *Sic sin. tot. ad sin. baseos* a b.
8	*Ex quovis lat.* a c, *& ang. ei adjac.* a.	*Ut sin. tot. ad sin. compl. ang. dati* a: *Sic tang. compl lat. dati* a c, *ad tang compl. basi* a b.
9	*Ex utroque latere* a c *& b c.*	*Ut sin. tot. ad sin. compl lat.* b c: *Sic sin. compl lat.* a c, *ad. sin. compl basi* a b.
10	*Ex omnibus angulis.*	*Ut tang. acuti* b, *ad tang. compl. acuti* a: *Sic sin. tot. ad sin. compl. basi.* a b.

Quæritur angulus aliquis *a* vel *b*.

11	*Ex latere oppos.* b c, *& altero acuto* b ♐ a.	*Ut sin. tot. ad sin. compl. lat. dati* b c: *Sic sin. ang. dati* b, *ad sin compl. ang.* a.
12	*Ex latere adjac* bc, *& altero acuto* a, ♐ b.	*Ut sin. compl. lat. dati* b c, *ad sin. tot.* *Sic sin. compl. ang. dati* a, *ad sin. ang. quæsiti* b.
13	*Ex basi* a b *& altero acuto* a, ♐ b.	*Ut sin. compl. basi* a b, *ad sin tot*: *Sic tang. compl. ang. dati* a, *ad tang ang.* b.
14	*Ex utroque latere* a c, b c.	*Ut sin. lat. adjac. quæsito, ad sinum tot*: *Sic tang. ulterius lat. ad tang. ang. quæsiti.*
15	*Ex latere oppos. & basi.*	*Ut sin baseos, ad sin. tot*: *Sic sin. lat. oppos. ad sin. ang. quæsiti.*
16	*Ex lat. adjac.* a c, *& basi* a b, ♐ a.	*Ut tang. compl lat. dati* a c, *ad tang compl. basi.* a b: *Sic sin. tot. ad sin. compl. ang.* a.

Ad



Ad laterum productionem, & quicquid schemate hoc explicatur, melius percipiendum plurimùm proderunt figuræ appositæ, in quibus arcus omnes triangulorum LEF in utrumque partem producuntur donec utrinque fiant quadrantes, adeoque & anguli E, X, R (*per. 7. c. 1. b*) recti. Notæ verò, & numeri casum quemlibet exprimentes, ita disponuntur, ut

1.° *prima Figura* continet omnes casus finuum ope resolubiles; *secunda* illos, ad quorum solutionem tangentes adhibentur. Unde primo intuitu scias, utrùm *per Regulam finuum*, an verò *tangentium*, ullus casus oblatus debeat resolvi.

2.° Notæ & numeri lateribus vel angulis appositi ostendunt, quid in unoquoque casu, & ex quibus datis quæratur; nam numerus casum exprimens semper adscribitur lateri vel angulo per illum casum quæsito: reliquæ verò notæ designant latera & angulos, ex quibus datis quæritur solutio. *vg.* in triangulo FZR (*in Fig.* 1.) numerus 9 adscriptus arcui seu basi FZ, indicat quæri in nono casu basim FZ: reliquæ verò notæ appositæ lateribus FR, ZR, & angulo recto R signant data, ex quibus in illo casu basis FZ quæritur. Similiter numerus 3 adhærens lateri ZR ostendit quòd *in* tertio casu quæratur latus aliquod ZR ex reliquis notatis; viz: ex basi FZ, latere FR, & angulo recto R. Quia verò utrumque casus habetur *in* 1. *Fig.* inde colliges utrumque solvi *per Regulam finuum*.

3.o Casus omnes, qui resolvuntur sine laterum productione, exprimuntur in triangulis intermediis LEF. Sic quia angulo ELF inscribitur (*in Fig.* 2) numerus 14. hinc inferes quod in casu decimo-quarto, *per Regulam tangentium* & absque laterum productione, quæratur aliquis angulus acutus ELF, ex lateribus notatis EL, EF, & angulo recto E notato.

4.o In cornibus XPL, FZR casus omnes arcuum protractione indigentes ita disponuntur, ut in triangulis intermediis semper habeatur propositi casûs solutio. *Ex. gr.* in triangulo XPL (*in Fig.* 1.) quæratur, ut in sexto casu, ex omnibus angulis notatis latus XL. 1.o Quia notatur uterque angulus acutus, productio fiet (*juxta Regulam* 3.) per angulum L adjacentem lateri notato XL. 2.o In triangulo LEF cognoscuntur angulus ELF æqualis dato XLP, & latus EF complementum anguli dati P. & angulus rectus E; ex quibus (*per Reg. sinuum*) invenitur basis LF, complementum lateris quæsiti XL.

CAPUT III.

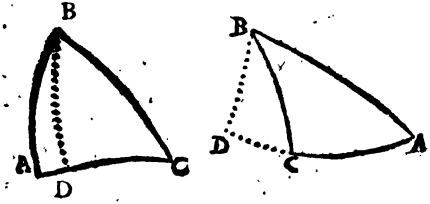
De analysi trianguli spharici obliquanguli.

PROPOSITIO I.

In omni triangulo spharico obliquangulo est
Ut sinus cujusvis lateris, ad sinum anguli sibi oppositi:
Sic sinus cujusvis alterius lateris, ad sinum anguli sibi oppositi.

Et è contrà.

Ut sinus cujusvis anguli, ad sinum lateris sibi oppositi:
Sic sinus cujusvis alterius anguli, ad sinum lateris sibi oppositi.



Hæc vocatur *Regula Sinuum* seu *Oppositionis*, & locum habet in triangulis obliquangulis, quandocunque invenitur Oppositio.

Demonstr. Ab aliquo angulo B ducatur arcus BD per

pendicularis ad latus oppositum AC, si opus fuerit productum. Sive BD cadat intra, sive extra triangulum ABC, erit in triangulo rectangulo ABD (*per.* 1. *c.* 1. *b*) sinus lateris AB ad sinum totum, ut sinus lateris BD ad sinum anguli A: & in triangulo rectangulo BCD, erit (*per eandem*) sinus totus ad sinum lateris BC, ut sinus anguli BCD ad sinum lateris BD. Ergo

1. *Sinus lat.* AB. 3. *Sinus ang.* BCD. (*ex æquo perturbato*) ut
2. *Sinus totus.* 1. *Sinus lat.* BD. sinus lateris AB, ad sinum
3. *Sinus lat.* BC. 2. *Sinus anguli* A. lateris BC; ita sinus anguli BCD, ad sinum anguli A. Et (*permutando*) ut sinus lateris AB ad sinum anguli BCD seu BCA (idem enim est utriusque sinus:) ita sinus lateris BC ad sinum anguli A; & (*invertendo*) ut sinus anguli BCA ad sinum lateris AB: ita sinus anguli A, ad sinum lateris CB.

PROPOSITIO II.

Resolvere triangulum sphæricum obliquangulum, in quo datur oppositio.

1.° Si quæratur angulus aliquis A, fiat

Ut sinus lateris dati AC, *ad sinum anguli oppositi* B:

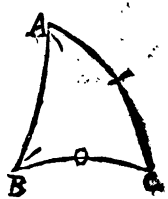
Sic sinus lateris noti BC, *ad sinum anguli quæsiti* A.

2.° Si quæratur latus aliquod BC, fiat

Ut sinus anguli dati B, *ad sinum lateris oppositi* AC.

Sic sinus anguli dati A, *ad sinum lateris quæsiti* BC.

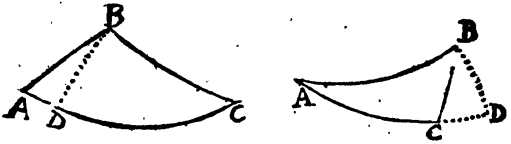
Ratio utriusque partis patet *ex præcedente.*



PROPOSITIO III.

Resolvere triangulum obliquangulum in quo nulla reperitur oppositio.

Quando nulla est oppositio, demitti debet ab aliquo angulo arcus perpendicularis, qui triangulum obliquangulum datum resolvat in duo triangula rectangula, quæ per regulas suprà traditas resoluta dabunt latera & angulos quæsitos.



3 *Ex lat.* a b, *&* 11.o *Quare* (ut in. 2. c.) a d. 2. d c, *dicendo*, *ang.* a *&* c *adjacen.* | *Ut tang. ang.* c, *ad tang. ang.* a: *lat. quæſit.* a c | *Sic ſin. lat.* a d, *ad ſin. lat.* d c.

Demonſtr. Ut tang. ang. *c*, ad tang. lat. *b d*:
Sic (*per.* 5. *c.*) ſinus totus, ad ſinum lat. *d c.*
Et, Ut tang. lat. *b d*, ad tang. ang. *a*:
Sic (*per.* 14. *c.*) ſinus lat. *a d*, ad ſinum totum.
Ergo (*per.* 23. *l.* 5) Ut tang. ang. *c*, ad tang ang. *a*:
Sic ſinus lat. *a d*, ad ſinum lat. *d c.*

4 *Ex lat.* a b, *&* 21.o *Quare* (ut in. 13. c.) *ang.* a b d. 2. b c, *dicendo*, *ang* a *&* b *adjac.* | *Ut ſin. compl. ang.* a b d, *ad tang. compl. lat.* a b: *dato lat.* a b, *&* b c. | *Sic ſin. compl. ang.* c b d, *ad tang. compl. lat.* b c.

Demonſtr. Ut ſinus totus, ad tang. compl. lat. *b d*:
Sic (*per.* 8. *c.*) ſinus compl. ang. *a b d*, ad tang. compl. lat. *a b.*
Et, Ut ſinus totus, ad tang. compl. lat. *b d*:
Sic (*per.* 8. *c.*) ſinus compl. ang. *c b d*, ad tang. compl. lat. *b c.*
Ergo (*per.* 11. *l.* 5) Ut ſinus compl. ang. *a b d*, ad tang. compl. lat. *a b*:
Sic ſinus compl. ang. *c b d*, ad tang. compl. lat. *b c.*

Angulus quæritur.

5 *Ex* 2 *ang* b *&* a, *&* 11.o *Quare* (ut in. 13. c.) *ang* a b d. 2. b c a, *dicendo*, *lat.* a b *ab eis in* | *Ut ſin. ang* a b d, *ad ſin. compl. ang.* a: *intercepto* 2 *c.* | *Sic ſin. ang.* c b d, *ad ſin. compl. ang.* c.

Demonſtr. Ut ſinus totus, ad ſinum compl. lat. *b d*:
Sic (*per.* 11. *c.*) ſinus ang. *a b d*, ad ſinum compl. ang. *a.*
Et, Ut ſinus totus, ad ſinum compl. lat. *b d.*
Sic (*per* 11. *c.*) ſinus ang. *c b d*, ad ſinum compl. ang. *c.*
Ergo (*per.* 11. *l.* 5) Ut ſinus ang. *a b d*, ad ſinum compl. ang. *a*:
Sic ſinus ang. *c b d*, ad ſinum compl. ang. *c.*

6 *Ex 2 ang.* a & c, & 1 l. o *Quare* (ut in. 13. c.) a b d. 2. a b c, *dicendo*, *lat.* a b *adjac. ang.* [*Ut sin compl. ang.* a, *ad sin. compl. ang.* c; *quæsit.* b. [*Sic sin. ang.* a b d, *ad sin. ang* c b d.

Demonstr. Ut sinus compl. lat. *b d*, ad sinum totum:
Sic (*per.* 12. *c.*) sinus compl. ang. *a*, ad sinum ang. *a b d.*
Et, Ut sinus compl. lat. *b d*, ad sinum totum:
Sic (*per.* 12. *c*) sinus compl. ang. *c*, ad sinum ang. *c b d.*
Ergo (*per.* 11. *l.* 5) Ut sinus compl. ang. *a*, ad sinum ang. *a b d.*
Sic sinus compl. ang. *c*, ad sinum ang. *c b d.*

7 *Ex 2 lat.* a b, a c, [1 l. o *Quare* (ut in. 2. c.) a d. 2. *ang.* c, *dicendo*, *& ang.* a *ab eis con-* [*Ut sin. lat.* c d, *ad sin. lat.* a d: *tento*, &c. [*Sic tang. ang.* a, *ad tang ang.* c.

Demonstr. Ut sinus lat. *c d*, ad sinum totum:
Sic (*per.* 14. *c.*) tang. lat. *b d*, ad tang. ang. *c.*
Et, Ut sinus totus, ad sinum lat. *a d:*
Sic (*per.* 4. *c.*) tang. ang. *a*, ad tang. lat. *b d.*
Ergo (*per.* 23. *l.* 5) Ut sinus lat. *c d*, ad sinum lat. *a d:*
Sic tang. ang. *a*, ad tang. ang. *c.*

8 *Ex ang.* a, *& 2 lat.* [1 l. o *Quare* (ut in. 13. c.) *ang.* a b d. 2. c b d, *dicendo*, a b, b c *comprehen-* [*Ut tang. compl. lat.* a b, *ad sin. compl. ang.* a b d; *dentib. ang. quæsit.* b. [*Sic tang compl lat.* b c, *ad sin. compl. ang.* c b d.

Demonst. Ut tang. compl. lat. *b d*, ad sinum totum:
Sic (*per.* 16. *c.*) tang. compl. lat. *a b*, ad sinum compl. ang. *a b d.*
Et, Ut tang. compl. lat. *b d*, ad sinum totum:
Sic (*per.* 16 *c.*) tang. compl. lat *b c*, ad sinum compl. ang. *c b d.*
Ergo (*per.* 11. *l.* 5) Ut tang. compl. lat. *a b*, ad sinum compl. ang. *a b d.*
Sic tang. compl. lat. *b c*, ad sinum compl. ang. *c b d.*

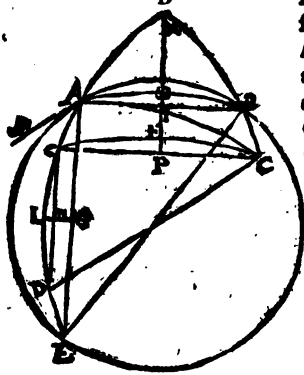
LEMMA AD PROP. IV.

In omni triangulo spharico, rectangulum sub sinubus crurum, est ad quadratum radii: ut rectangulum sub sinubus differentiarum crurum à semisumma omnium laterum, Ad quadratum sinûs dimidii anguli verticalis.

Proponatur triangulum BCA; sit angulus quæsitus & verticalis B; crus minus BA, crus majus BC; basis AC.

CONSTRUCTIO.

Ex crure majori BC auferatur B *a* arcus æqualis cruri minori BA, & producatur BA ad *c*, ut adæquet BC. Puncta A *a* & *c* C jungantur arcubus circulorum maximorum AO *a* & *c* FC, quibus subtendantur chordæ AI *a*, *c* PC: angulum verticalem B bisecet planum circuli maximi BOF, bisecabit (*per* 16. *c.* 1. *b*) etiam arcus AO *a*, *c* FC & eorum chordas.



Dein trianguli basis AC transferatur in circulum BA *c* ex A usque ad D, & ex *c* usque ad E, subtensis chordis AE, *c* D, quæ simul cum arcubus, quos subtendunt, bisecentur per rectam LNG. Ducantur etiam rectæ E *a*, DG & huic parallela MA. Denique per puncta EA *a* describatur circulus. In hac constructione

1.° Chordæ A *a*, *c* C (*per* 16. *c.* 2. *b*) sunt parallelæ, sicut & AE, *c* D (*per* 1. *Corol. p.* 29. *l.* 3.)

2.° Arcus BL est semissumma omnium laterum; nam crus minus AB duplicatur per superimpositionem majoris BC; & residuum cruris majoris BC, viz: A *c*, duplicatur per superimpositionem basis AC ex A in D; & residuum basis AC, viz: *c* D, duplicatur in ejus dimidio *c* L.

3.° Arcus AL est differentia inter crus minus BA & BL semissummam omnium laterum. Et AG est sinus illius differentiæ; nam (*per Lemma ante* 16. *c.* 1 *b*) est perpendicularis lineæ LNG quæ à centro ducta bisecat arcum ALE, ejusque chordam AGE.

Similiter arcus *c* L est differentia inter crus majus BC (seu B *c*) & BL semissummam omnium laterum, & *c* N est sinus illius differentiæ.

4.° Recta I *a* est sinus arcus O *a*, & PC sinus arcus FC; nam

nam (*per Lemma ante 16. c. 1. b*) sunt perpendiculares dimetris bisecantibus arcus AO *a*, *c*FC in O & F.

5.° Angulus X est dimidium anguli verticalis ABC, quippe quem (*ex constr.*) bisecat arcus BOP.

Ostendi igitur debet rectangulum sub sinubus crurum BA, BC, esse ad quadratum radii, ut rectangulum sub AG & *c*N, est ad quadratum sinûs anguli X.

DEMONSTRATIO.

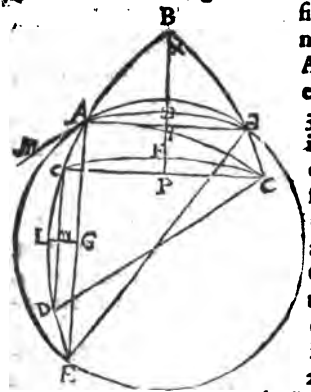
In triangulo sphærico B *a* O, quod (*ut patet ex 8 & 9 16. c. 1. b*) est rectangulum in O, sinus baseos B *a* est (*per 1. & 2. b*) ad sinum totum: ut I *a* sinus lateris O *a*, ad sinum anguli oppositi X.

Similiter in triangulo sphærico rectangulo BCF sinus baseos BC est ad sinum totum, ut PC sinus lateris FC ad sinum anguli oppositi X.

Multiplica nunc inter se utriusque analogiæ terminos, & erit (per *Corol. p. 22. l. 5*) rectangulum sub sinubus crurum B *a* (seu BA) & BC, ad quadratum sinûs totius, ut rectangulum sub I *a*, PC, ad quadratum sinûs anguli X.

Quòd primus, secundus, & quartus terminus hujus analogiæ iidem sint, qui in Lemmate propositi, patet *ex c. constr.* Quòd autem tertius æquetur tertio, id est, rectangulum sub I *a*, PC, rectangulo sub AG, *c*N, sic ostendo.

Quia parallelæ sunt A *a* & *c* C, item AE & *c* D, æquales sunt (*per 10. l. 11.*) anguli *a*AE & C *c* D, & (*per 15. l. 11.*) plana triangulorum AE *a*, CD *c* per istas lineas ducta sunt parallela. Deinde quia triangulum sphæricum ADC est (*ex constr.*) Isoscele, recta MA (*ex constr.*) parallela chordæ DC (*per 15. c. 1. b*) tangit sphæram in puncto A; & quia tangit planum trianguli AE *a* in puncto A, & parallela est chordæ DC existenti in plano *c*CD parallelo priori AE *a*, tota MA (*per Corol. p. 16. l. 11.*) est in eodem plano cum triangulo AE *a*: ergo, quia triangulum AE *a* est in eodem plano cum circulo AE *a*, MA est etiam in eodem plano: ergo MA (*per 15. c. 1.*) tangens sphæram in A, tangit etiam circulum AE *a* in eodem puncto A; cùm enim tota circumferentia circuli AE *a* sit (*per 2. Corol. p. 1. c. 1. b*) in super-

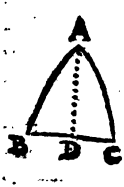


ficie ſphæræ, patet quòd MA non poſſit ſecare circulum AE *a*, quin ſecet ſphæram: ergo angulus MAE eſt (*per* 32. *l.* 3) æqualis angulo A *a* E in ſegmento alterno. Sed quia MA, DC, & AE, *c*D ſunt parallelæ, anguli MAE, CD*c* (*per* 10. *l.* 11) ſunt æquales: ergo anguli A *a* E, CD*c* ſunt æquales; adeoque in triangulis AE *a*, C *c* D æquales ſunt anguli *a* AE, C *c* D, item A *a* E, CD*c*: ergo (*per* 2. *Corol. p.* 4. *l.* 6) A*a*: cD :: AE: *c* C. Ergo dimidiando (*per* 15. *l.* 5) I *a*: *c* N :: AG: PC: ergo (*per* 16. *l.* 6) rectangulum ſub extremis I *a*, PC, æquatur rectangulo ſub mediis AG, *c*N.

PROPOSITIO IV.

Datis tribus lateribus invenire angulos.

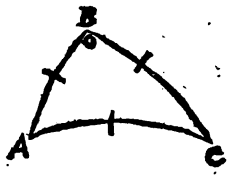
1.o Si latus aliquod AB ſit quadrans, aliud latus AC vel AE producatur vel minuatur ad quadrantem AD, & ducatur arcus BD. In triangulo CBD, vel EBD ſcitur latus CD vel DE differentia dati AC vel AE à quadrante AD, & latus BC vel BE datum: angulus verò D (ſicut & ABD) eſt rectus, (ut oſtendam *prop.* 6. *c.* 4. *h.*) Ex his invenietur arcus BD, qui (*per* 3. *c.* 1. *h*) eſt menſura anguli A.



2.o Si proponatur triangulum Iſoſcele ABC, ab angulo A inter æqualia latera contento, ad baſim BC biſectam in D, ducatur arcus AD. Jam quia triangula ABD, ACD ſunt (*ut patet*) ſibi mutuò æquilatera, anguli ad D ſunt (*per* 14. *c.* 1. *h*) æquales; adeoque (*per* 2. *c.* 1. *h*) recti. Ergo in triangulo ABD cognoſcuntur la-

tera AB, BD cum angulo recto D. Ex his (*per* 3. *c.* 2. *h*) noti fient anguli B & BAD, quibus (*per* 14. *c.* 1. *h*) æquantur anguli C & CAD.

3.° Extra casus prædictos, fiat (si quæratur *v. g.* angulus B) ut rectangulum sub sinubus crurum AB, CB, ad quadratum radii: sic rectangulum sub sinubus differentiarum crurum AB, CB à semisumma omnium laterum, ad aliud; & erit (*per Lemma*) quartus terminus quadratum sinûs dimidii anguli quæsiti B.



Nota angulum quæsitum semper accipiendum esse pro angulo verticali, & latus oppositum pro basi.

Ut meliùs intelligatur casûs hujus solutio, praxim tum per sinus naturales, tum per Logarithmos exhibebo. Sit igitur in triangulo proposito ABC crus minus BA 39. gr. 20′. crus majus BC 60 gr. basis AC 78 gr. 30′.

Ordo resolutionis per sinus naturales.

1.° Multiplicando inter se sinus naturales crurum BA BC invenitur rectangulum sub sinubus crurum, quod est primus terminus.

2.° Ducendo sinum totum in seipsum, habetur quadratum radii, quod est secundus terminus.

3.° Subtracto crure BA ex semisumma omnium laterum; quære sinum residui seu differentiæ: similiter subtracto crure BC ex eadem semisumma, quære sinum differentiæ. Hos differentiarum sinus inter se multiplica, & prodibit rectangulum sub sinubus differentiarum crurum à semisumma omnium laterum, quod est tertius terminus.

4.° Ducatur secundus terminus in tertium, & productus dividatur per primum; erit quotiens quadratum sinûs dimidii anguli B; hujus radicem extrahe, notusque erit dimidius, adeoque totus angulus B.

Forma calculi per Logarithmos.

1.o Sinus Logarithmicos { lat: BA --- 98019735. & lat: BC -- 99375306. } Adde.
Erit ſumma --- 197395041.

Rectangulum ſub ſinubus crurum, & primus terminus.

2.o Duplica Logarithmum ſinûs totius --- 100000000.
ut habeas quadratum radii ------ 200000000.
quod eſt, ſecundus terminus.

3.o Latera { BA. 39 gr. -- 20′ / BC 60 --- 00 / AC 78 --- 30 } Adde ut habeatur

Laterum ſumma. 177 --- 50 &
Semiſumma ---- 88 --- 55 hinc ſubtrahe
Latus BA --- 39 --- 20
Reſiduum --- 49 --- 35 eſt prima differentia.
Ex eadem ſemis. 88 --- 55 ſubtrahe
Latus BC --- 60 --- 00
Reſiduum --- 28 --- 55 eſt ſecunda differentia.

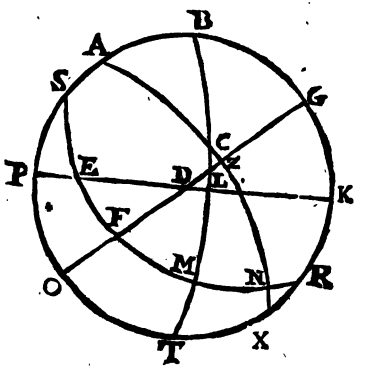
Differentiarum Logarithmicos ſinus { 98815842 / 96844297 } Adde
Erit ſumma --- 195660139 Rectangulum ſub ſinubus differentiarum, quod eſt tertius terminus.

4.o 2 Terminum --- 200000000 } Adde
& 3 Terminum 195660139 }
Ex ſumma --- 395660139 } Subtrahe
1 Terminum 197395041 }
Reſiduum ---- 198265098 eſt quadratum ſinûs dimidii anguli B; ac proinde radix ſeu ſemiſſis ejus 99132549 eſt ipſe ſinus, qui in tabulis inventus dat 54 gr. 59′, pro dimidio anguli B; adeoque totus angulus B continet 109 gr. 58′.

LEMMA AD PROP. V.

Dato quacunque triangulo ſpherico (ABC) *eſt in ſphæra aliud triangulum* (DEF) *cujus ſingula latera* (DF, DE, EF) *æquantur ſingulis angulis* (A, B, C:) *& ſinguli anguli* (D, E, F,) *ſingulis lateribus* (AB, BC, AC:) *mutato tamen utrinque angulo maximo v. g.* B & F *in ſuum complementum ad duos rectos.*

Perficiatur circulus AB, cui occurrant producti AC in X, & BC in T. Deinde punctis A, B, C, tanquam polis seu centris, intervallo quadrantis describantur circuli GO, KP, RS. Hinc



1.o Quia A est polus circuli GO, anguli G, Z (*per 6. c. 1. h*) sunt recti: similiter quia B est polus circuli KP, recti sunt anguli K, L; & quia C est polus circuli RS, recti sunt anguli M, N.

2.o Quia anguli G & K sunt recti, arcus GD, KD transeunt (*per 6. c. 1. h*) per polum circuli GK, qui proinde est D, adeoque (*per 5. c. 1.*) GD, KD sunt quadrantes. Similiter quia recti sunt anguli Z, N, arcus ZF, NF concurrunt in polo F, & sunt quadrantes. Denique quia anguli L & M sunt recti, arcus ME, LE concurrunt in polo E, & sunt quadrantes.

Demonstr. jam I. Pars. Et 1.o Quia quadrantes sunt LE, KD, sublato communi DL, restant DE, KL æquales. Sed (*per 3. c. 1. h*) KL, adeoque & DE, est mensura anguli KBL, qui complet angulum maximum B ad duos rectos. Ergo.

2.o Quia quadrantes sunt ZF, GD, sublato communi ZD, restat DF æqualis arcui GZ, qui est (*per 3. c. 1. h*) mensura anguli BAC.

3.o Quia quadrantes sunt ME, NF, sublato communi MF, restat EF æqualis arcui MN, qui est mensura anguli MCN, id est (*per 2. c. 1. h*) anguli ACB.

2. *Pars.* 1.o Quia quadrantes sunt CN, AZ, sublato communi CZ, restat AC æqualis arcui ZN, mensuræ anguli ZFN, qui complet angulum maximum EFD ad duos rectos.

2.o Quia quadrantes sunt CM & BL, ablato communi CL, restat LM mensura anguli FED, & æqualis arcui BC.

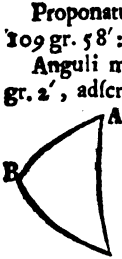
3.o Quia AG & BK sunt quadrantes, subtracto communi BG, restat AB æqualis arcui GK, qui est mensura anguli GDK, id est (*per* 2. *v.* 1. *h*) anguli EDF.

PROPOSITIO V.

Datis tribus angulis invenire latera.

Proponatur triangulum ABC, in quo angulus B sit *v. g.* 109 gr. 58′: A 56 gr. 10′. C 37 gr. 26′.

Anguli maximi B complementum ad duos rectos, viz: 70 gr. 2′, adscribe lateri oppositi AC: lateri BC appone 56 gr. 10′. quantitatem anguli oppositi A: & lateri AB 37 gr. 26′. quantitatem anguli oppositi C. Angulis sic in latera opposita translatis, (*per præcad.*) ex tribus lateribus datis invenies angulum B esse 101 gr. 30′. Accipe ejus complementum ad duos rectos, viz: 78 gr. 30′. erit hæc vera quantitas lateris oppositi AC.



Nota quòd invento latere ex tribus angulis, vel uno angulo ex tribus lateribus, reliqua facilè investigentur per Regulam sinuum.

CAPUT IV.

De specie laterum & angulorum.

Quia (ut alibi diximus) arcus & anguli eosdem habent sinus cum suis complementis ad duos rectos; hinc fit ut, invento per trianguli analysim sinu aliquo, dubitari possit, qualis ipsi arcus vel angulus correspondeat: *v. g.* invento sinu 87 gr. dubitari potest utrùm angulus illi debitus sit 87 gr. an verò 93 gr. prioris complementum ad duos rectos. Ut huic incommodo occurratur, propositiones aliquas subjiciam, quæ ad determinandam laterum & angulorum speciem pro regulis erunt.

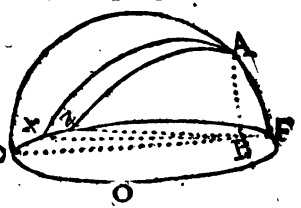
Consultius fortè erit ut caput hoc à Tyronibus initio prætermittatur, quia laterum & angulorum species plerunque potest aliunde determinari.

PROPOSITIO I.

Si ex puncto (A) *quod non est polus circuli* (DOEZ) *ad*

ejus circumferentiam ducantur circulorum maximorum arcus quotcunque (AD, AX, AZ;) *arcus* (AD) *per polum transiens eſt maximus: & reliquis ille major qui maxime vicinior.*

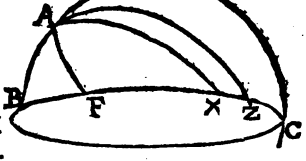
Demonſtr. Quia arcus DAF tranſit per polum circuli DOFZ, (*per 6. c. 1. h*) illi inſiſtit ad angulos rectos: ergo planum DAF (*per 4. def. c. 1. h*) eſt rectum plano DOFZ: ergo ſi ducatur AB perpendiculatis ad ſectionem communem DF, erit AB (*per 4. def. l. 11.*) recta plano DOFZ; adeoque (*per 3. def. l. 11.*) etiam lineis BD, BX, BZ: ergo triangula ABD, ABX, ABZ (ſubintelligi debent in hac figura chordæ arcuum AD, AX, AZ, quæ ad vitandam confuſionem non exprimuntur) ſunt rectangula in B: ergo cùm latus AB ſit commune, & BD per centrum tranſiens ſit (*per 7. l. 3.*) majus quàm BX, & BX quàm BZ; baſis AD (*per 5. Corol. p. 47. l. 1.*) major eſt baſi AX, & AX major baſi AZ: ergo (*per 26. l. 3.*) arcus AD major eſt arcu AX, & AX major arcu AZ.



PROPOSITIO II.

In triangulo ſpherico rectangulo, quodvis latus circa angulum rectum eſt ejuſdem ſpeciei cum angulo oppoſito.

Hoc eſt, ſi latus aliquod BX circa angulum rectum B ſit quadrans, angulus oppoſitus BAX erit rectus: ſi ſit majus minuſve quadrante, angulus erit ſimiliter major minorve recto: & è converſo.



Demonſtr. 1.° Quia angulus B eſt rectus, & BX quadrans; eſt X (*per 5. & 6. c. 1. h*) polus circuli BA. Ergo quia AX tranſit per polum, angulus BAX (*per 6. c. 1. h*) eſt rectus.

2.° Si ſit BF minus quadrante, baſis AF cadet inter X & B: ergo angulus BAF erit minor recto BAX.

3.o Si sit BZ majus quadrante, basis AZ cadet ultra AX: ergo angulus BAZ est major recto BAX.

Conversa patet; nam si anguli differrent specie à lateribus oppositis, latera non essent ejusdem speciei cum angulis oppositis.

PROPOSITIO III.

Si latera circa angulum rectum, vel anguli ad basim, sint ejusdem speciei, & neque quadrantes, neque recti; basis est minor: si sint diversa, est major quadrante.

Demonstr. I. Pars. Sint 1.o BA, BF minora quadrante: ergo (*per* 5. *c.* 1. *h*) A non est polus circuli BX: ergo (*per* 1. *h*) basis AF est minor quadrante AX.

2.o Sint circa G angulum rectum, CA, CF majora quadrante: A (*per* 5. *c.* 1. *h*) non erit polus circuli CXB: ergo (*per* 1. *h*) basis AF est minor quadrante AX, nam magis distat ab AC transeunte per polum.

2. *Pars.* Sit latus CA majus, & CZ minus quadrante. Quia CA est majus, A non est polus circuli CXB: & quia CZ est minus quadrante, basis AZ cadet inter C & polum X: ergo (*per* 1. *h*) est major quadrante AX.

Quod dictum de lateribus, verum est de angulis ad basim; quia sunt ejusdem speciei cum lateribus oppositis.

PROPOSITIO IV.

Si basis sit major quadrante, latera sunt diversa speciei: si minor, sunt ejusdem.

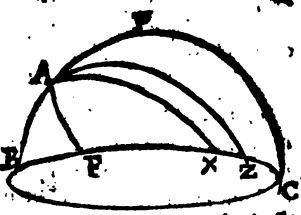
Patet ex præced: nam si latera essent ejusdem speciei, basis esset minor: ergo quando est major quadrante, sunt speciei diversæ. Similiter si essent diversæ speciei, basis esset major: ergo quando basis est minor, sunt ejusdem.

PROPOSITIO V.

Si latus aliquod (BX) *circa angulum rectum* (B) *sit quadrans, etiam basis* (AX) *est quadrans: & è converso.*

Demonstr. I. Pars. Quia angulus B est rectus, & BX quadrans, punctum X (*per* 5. & 6. *c.* 1. *h*) est polus circuli BA: ergo

ergo (*per* 5. *c.* 1. *h*) basis AX est quadrans.



2. *Pars.* Si neutrum latus sit quadrans, sunt vel singula majora, vel singula minora, vel unum majus, alterum minus quadrante: sed in primo & secundo casu basis (*per* 3. *h*) est minor, & in tertio est major quadrante: ergo quando basis est quadrans, aliquod aliud latus est quadrans.

Corollarium.

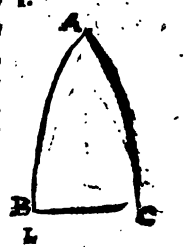
In omni triangulo rectangulo vel duo tantùm latera majora sunt quadrante, vel nullum est majus; quòd enim non possint esse tria singula quadrante majora, statim ostendam. Unde quotiescunque in triangulo rectangulo datur unum latus majus quadrante, produc latera duo majora donec fiant semicirculi, & habebis (*per* 2. *Corol. p.* 4. *c.* 1. *h*) novum triangulum rectangulum, cujus singula latera erunt quadrante minora.

Propositiones explicatæ sufficiunt ad determinandam laterum & angulorum speciem, si duos casus excipias. Primus est quando vel duo anguli sunt recti; vel duo latera quadrantes; nam ex his nil potest de tertio angulo vel latere determinari. Secundus habetur, quando solùm cognoscitur latus aliquod circa angulum rectum, cum angulo ipsi opposito; nam possunt hæc esse ejusdem speciei in triangulis diversis, ut patebit consideranti triangula ACZ, ACX, ACF, item ABF, ABX, ABZ.

Quomodò & quatenùs hæc possint applicari triangulis obliquangulis posteà ostendam.

PROPOSITIO VI.

Si in triangulo spherico Isosceli (ABC) *latera æqualia* (AB, AC) *sint quadrantes, anguli* (B & C) *ad basim sunt recti; si sint majora, anguli sunt obtusi; si minora, sunt acuti.*



Demonstr. Quia latera AB, AC æquantur semicirculo, anguli B & C (*per* 13. *c.* 1. *h*) simul æquant duos rectos: ergo cùm (*per* 11. *a.* 1. *h*) sint æquales, ambo sunt recti.

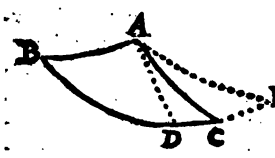
Eodem modo (*per* 11. & 13. *c.* 1. *h*) demonstrantur secunda & tertia pars propositionis.

Corollarium.

Eodem ferè modo (*per conversas* 11 & 13. *c.* 1. *h*) demonstres, quòd si anguli ad basim sint ambo recti, latera AB, AC sint quadrantes: item quòd si anguli isti sint inter se æquales & simul majores duobus rectis, latera opposita AB, AC sint singula quadrante majora: minora verò, si anguli æquales B & C sint acuti.

LEMMA.

Si anguli (B, C) *ad basim sint obliqui & ejusdem speciei, perpendiculum ab alio angulo* (A) *ductum cadet intra: aliàs cadet extra triangulum.*



Demonstr. I. Pars. Quia si caderet extra, ad E *v. g.* anguli ABE, ACE (*per* 2. *h*) convenirent specie cum latere opposito AE; adeoque & inter se. Sed (*ex suppos.*) ABE, ACB sunt ejusdem speciei: ergo ACB, ACE sunt ejusdem speciei; quod est absurdum, cùm ACB ponatur obliquus, & ACE sit (*per* 2. *c.* 1. *h*) ejus complementum ad duos rectos.

2. *Pars.* Quia si perpendiculum AD cadat intra triangulum, anguli B & ACB erunt (*per* 2. *h*) ejusdem speciei cum latere opposito AD: ergo non different specie inter se.

PROPOSITIO VII.

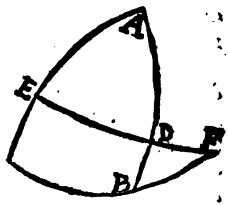
Si alicujus trianguli sphærici (ABC) *omnes anguli sint acuti, singula latera sunt minora quadrante.*

Demonstr. Ab angulo A demittatur perpendiculum AD. Quia anguli B, C sunt (*ex suppos.*) acuti, adeoque ejusdem speciei, AD cadet intra triangulum, & erunt in triangulo ADC ad basim AC anguli C & DAC ambo acuti: ergo (*per* 3. *h*) basis AC est minor quadrante. Si ab angulis B & C duxeris perpendi-

cula, idem eodem modo demonstres de lateribus AB, BC.

PROPOSITIO VIII.

Si trianguli sphærici (ABC) *singula latera sint quadrante majora, omnes anguli sunt obtusi.*

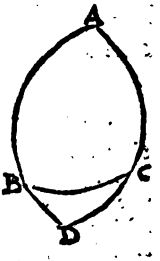


Demonstr. Si omnes anguli essent acuti, omnia latera (*per 7 h*) essent minora quadrante: ergo aliquis est vel rectus, vel obtusus. Sit ille angulus A. Centro A, intervallo quadrantis, describe arcum ED concurrentem in F cum CB producto. Erit A (*per 5. c. 1. h*) polus circuli ED, & arcus AE, AD quadrantes: ergo angulus AED, adeoque (*per 2. c. 1. h*) & CEF est rectus. Item ED est (*per 3. c. 1. h*) mensura anguli A vel recti, vel obtusi. Ergo ED est ad minimum quadrans, adeoque EF est major quadrante. Sed etiam arcus CB (*ex suppos.*) adeoque & CF est major quadrante: ergo CF, EF simul excedunt semicirculum: ergo (*per 13. c. 1. h*) anguli ECF, CEF simul excedunt duos rectos. Sed (*ex dictis*) CEF est rectus: ergo ECF seu angulus C est obtusus. Eodem modo, si produxeris DE, BC donec concurrant versùs E & C, ostendas angulum B obtusum: idemque similiter ostendetur de angulo A.

Corollaria.

1. *Eodem ferè modò ostendas omnes angulos esse obtusos, si sint duo latera* AC, AB *majora quadrante, & tertium* CB *quadrans.*

2. *Si in triangulo aliquo duo latera* BD, CD *sint minora quadrante, & tertium* BC *majus; angulus maximo lateri oppositus, est obtusus, & duo reliqui acuti.*



Producantur DB, DC, donec concurrunt in A, erunt (*per 2. Corol. p. 4. c. 1. h*) trianguli ABC latera omnia majora quadrante, adeoque omnes anguli obtusi. Sed anguli A & D sunt æquales: ergo D est obtusus. Anguli verò DBC, DCB sunt (*per 2. c. 1. h*)

complementa obtusorum ACB, ABC, adeoque acuti.

3. *Si in triangulo* ABC, *duo anguli quivis* B *&* C *sint obtusi, aliqua duo latera sunt majora quadrante.*

Nam si tertius A sit acutus, trianguli oppositi DBC anguli omnes erunt acuti: ergo (*per* 7. *h*) latera DB, DC sunt minora, adeoque AB, AC sunt majora quadrante.

Si verò omnes anguli sint obtusi; quia A & B sunt majores duobus rectis: arcus AC, BC sunt simul (*per* 13. *c.* 1. *h*) majores semicirculo: ergo aliquis eorum est major quadrante. Eodem modo ostendam AC & AB, item BC & AB excedere semicirculum: ergo aliqua duo latera singula excedunt quadrantem.

4. *Si in triangulo obliquangulo sint duo latera minora quadrante, duo anguli sunt acuti.*

Nam si duo essent obtusi, duo latera (*per præced.*) essent majora quadrante.

5. *Si in triangulo obliquangulo aliquod unum latus sit majus quadrante, aliquis angulus est obtusus.*

Nam si omnes anguli essent acuti, omnia latera (*per* 7. *h*) essent minora quadrante.

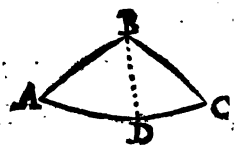
6. *Si nullus angulus sit acutus, latus aliquod est minus quadrante.*

Nam si omnia essent majora, omnes anguli (*per* 8. *h*) essent obtusi. Si omnia essent quadrantes, omnes anguli (*per* 3. *c.* 1. *h*) essent recti. Si duo essent quadrantes & tertium latus majus quadrante; anguli quadrantibus oppositi (*per* 11. *&* 13. *c.* 1. *h*) essent recti, angulus verò majori lateri oppositus (*per* 3. *c.* 1. *h*) major recto. Demum si duo latera essent quadrante majora & unum quadrans, omnes anguli (*per* 1. *Corol. h*) essent obtusi.

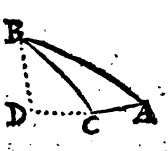
PROPOSITIO IX.

In triangulis obliquangulis laterum & angulorum species determinare.

Si in triangulo proposito dentur tria latera, vel tres anguli, præsens difficultas locum non habet; nam (*per* 4. *&* 5. *c.* 3. *h*) invenitur semissis anguli verticalis, quæ semper est (*per* 10. *c.* 1. *h*) minor recto. Tota igitur difficultas reducitur ad alios quatuor casus quos hîc breviter explicabimus.



PRIMUS CASUS.



Quando dantur duo latera (BA, AC) *& angulus contentus* (A,) *reliquorum ſpeciem invenire.*

Ducatur perpendiculum BD. In triangulo rectangulo BAD, ex angulo A (*per* 2. *h*) cognoſcitur ſpecies lateris BD. Ex BA baſi & latere BD (*per* 4. *h*) habetur ſpecies lateris AD. Inveniatur (*per* 3. *c.* 2. *h*) latus AD, & notum erit latus DC. Ergo in triangulo BDC habetur ſpecies laterum BD, DC; adeoque (*per* 2. *h*) & angulorum C & DBC, & [*per* 3. *h*] baſeos BC.

SECUNDUS CASUS.

Quando dantur duo anguli [A & B] *cum latere interjecto* [BA] *reliquorum ſpeciem determinare.*

Ab aliquo angulo noto [B] demittatur perpendiculum BD. In triangulo rectangulo BAD, ex angulo A [*per* 2. *h*] ſcitur ſpecies lateris BD. Ex ſpecie lateris BD & baſeos BA [*per* 4. *h*] cognoſcitur ſpecies lateris AD. Invento [*per* 3. *c.* 2. *h*] angulo ABD, habetur angulus CBD, adeoque [*per* 2. *h*] ſpecies lateris DC. Ergo in triangulo DBC ſcitur ſpecies laterum BD, DC; adeoque [*per* 2. *h*] angulorum C & CBD, [*per* 4. *h*] baſeos BC.

TERTIUS & QUARTUS CASUS.

Notis duobus lateribus cum angulo eorum uni oppoſito; vel duobus angulis cum latere quod opponitur alicui angulo noto, cæterorum ſpeciem determinare.

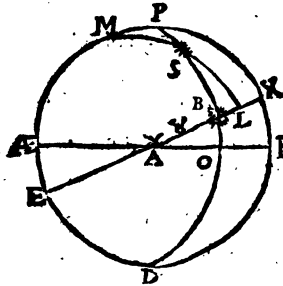
Pro his duobus caſibus non eſt aliqua regula ſeu methodus generalis, per quam reliquorum arcuum & angulorum ſpecies poſſit univerſaliter determinari. In caſibus particularibus ad hoc inſervient *Corollaria poſt Prop. præced.* & cap. I. *Prop.* 9. 10. 11. 12. 13. quæ Lector conſulat.

Nota non omnes Geographos in eodem loco constituere primum Meridianum. Angli & Belgæ à Monte Teneriffa, Galli à parte occidentali Insulæ de *Ferro*, alii ab aliis locis longitudinum initium sumunt. Unde contingit in designanda locorum longitudine magna inter Authores discrepantia. Observari igitur debet inter legendum, quod longitudinis initium unusquisque Author statuerit.

PROBLEMA I.

Cognitâ maximâ declinatione Solis (quæ est gr. 23. 30.') & ejus distantiâ à proximo puncto Æquinoctiali, invenire præsentem declinationem: & è converso, datâ præsenti Solis declinatione, & declinatione maximâ, invenire ejus locum in Ecliptica.

Sit in Ecliptica EX locus Solis B, distans 60 gr. ab initio Arietis A. In triangulo BOA, angulus O (*per 6. c. 1. h*) est rectus, nam declinationis circulus PBD (*per 8. def. h*) transit per P, D Polos Æquatoris ÆR: angulus A (*per 3. c. 1. h*) eosdem gradus continet quos XR maxima Solis declinatio, viz. 23. gr. 30'. Et latus AB (*ex suppos.*) est 60. gr. Ex his tribus (*ut in casu primo schematis c. 2. h*) invenitur præsens Solis declinatio OB 20 gr. 12'.



E converso: Si dentur anguli O rectus, & A 23 gr. 30' cum præsenti declinatione Solis OB 20 gr. 12'. Ex his tribus (*ut in casu 7. ejusdem schem.*) invenitur Solis in Ecliptica locus B distare 60 gr. ab initio Arietis A, id est, arcum BA esse 60 gr.

PROBLEMA II.

Iisdem cognitis invenire ascensionem rectam Solis aut alterius puncti Ecliptica: & è converso, his datis invenire Solis declinationem.

Sit B locus Solis vel puncti in Ecliptica, cujus quæritur ascensio

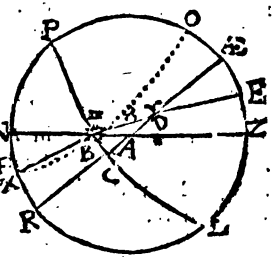
aſcenſio recta AO. In triangulo AOB angulus A eſt (*ut priùs*) 23 gr. 30′ : angulus O eſt rectus, quia (*per def. 9. b*) in ſphæra recta Æquator ÆR eſt rectus Horizonti PBD : latus verò AB ſupponitur notum, *v. g.* 60 gr. quale eſt quando Sol eſt in initio Geminorum. Ex his tribus (*ut in 2. caſu 1. ſchem.*) eruitur latus AO 57. gr. 48′. Aſcenſio recta puncti B.

Et è converſo, datâ aſcenſione rectâ AO, cum angulis O & A, invenitur (*ut in 4. caſu 1. ſchem.*) BO Solis declinatio.

PROBLEMA III.

Datâ Poli altitudine cum declinatione Solis invenire differentiam aſcenſionalem, & conſequenter aſcenſionem obliquam: & è converſo, datâ differentiâ aſcenſionali invenire Solis declinationem.

Sit in EX, Ecliptica, B locus Solis orientis in Horizonte obliquo ZN ; ſit etiam æquator ÆR ejuſque polus P. Horizon rectus & circulus declinationis Solis ſit PCL ; erit aſcenſio recta DC, aſcenſio obliqua DA, differentia aſcenſionalis AC. In triangulo ABC, latus BC eſt declinatio Solis data, *v. g.* gr. 20. 12′ : angulus C (*per 6. c. 1. b*) eſt rectus : angulus A æqualis eſt arcui NR diſtantiæ Horizontis ZN ab Æquatore ÆR : quæ diſtantia eſt complementum altitudinis poli, ut patebit intuenti ſphæram Armillarem. Cùm igitur hîc Leodii altitudo poli ſit 50 gr. 40′, erit ejus complementum, adeoque & angulus A 39 gr. 20′. Ex his tribus (*ut in 5. caſu 1. ſchem.*) invenitur latus AC differentia aſcenſionalis 26 gr. 41′. Hanc ſubtrahe ex DC aſcenſione rectâ 57 gr. 48′. priùs inventâ, reſiduum 31 gr. 7′, erit aſcenſio obliqua DA.



E converſo. In triangulo ABC notâ differentiâ aſcenſionali AC, & angulis A & C, invenitur (*ut in 4. caſu 1. ſchem.*) declinatio Solis BC.

Nota, quando Sol Borealis eſt, differentiam aſcenſionalem

ſubtrahendam eſſe aſcenſioni rectæ, ut habeatur aſcenſio obliqua: addendam verò, ut habeatur deſcenſio obliqua. Quando autem Sol Auſtralis eſt, invenitur aſcenſio obliqua addendo, & deſcenſio obliqua ſubtrahendo aſcenſioni rectæ differentiam aſcenſionalem.

Corollarium.

Notâ Solis differentiâ aſcenſionali determinare horam Ortûs vel Occaſûs Solaris.

Quamvis Solis motus reverâ ſpiralis ſit; Aſtronomi tamen, calculi commoditate ducti, imaginantur Solem motu diurno deſcribere circulum vel in æquatore vel æquatori parallelum; cujus portio illa quæ ſupra Horizontem extat, vocatur *Arcus diurnus*, eſtque in Æquinoctiis ſemicirculus æquatoris, extra Æquinoctia aliquando major, interdum minor ſemicirculo. Unde ſequitur arcum ſemidiurnum nunc excedere, nunc deficere à quadrante. Ut igitur pro die conſtituto inquiratur ortus Solis, inveniri debet 1.° quantus ſit arcus ſemidiurnus, quem Sol tunc deſcribit. 2.° ſpatium temporis iſti Solis motui correſpondens.

Sit PRÆ Circulus Meridianus, OF arcus diurnus, reliqua conſtructio ut in *Probl.* 3. Jam quia Solis exiſtentis in B aſcenſio recta eſt (*ex dictis*) DC, & aſcenſio obliqua DA; erit differentia aſcenſionalis CA: & quia Sol oritur in B, erit BO arcus ſemidiurnus, qui ſimilis eſt æquatoris arcui ÇÆ. Sed differentia inter arcum CÆ & quadrantem AÆ eſt aſcenſionalis differentia CA. Cùm igitur, Sole exiſtente in ſignis borealibus, arcus ſemidiurni ſint in Hæmiſphærio boreali quadrante majores; minores verò, dum Sol eſt in Auſtralibus; ſequitur quòd differentia aſcenſionalis, quando Sol Borealis eſt, addi debeat quadranti, ut habeatur arcus ſemidiurnus; ſubtrahi verò, quando eſt Auſtralis. Unde addendo quadranti differentiam aſcenſionalem 26. gr. 41′. ſuprà (*Probl.* 3.) inventam, habebitur arcus ſemidiurnus 116. gr. 41′. cùm Sol eſt in initio Geminorum.

Deinde, quia ſingulis 24 horis æquator & circuli omnes ei paralleli toti tranſeunt per Meridianum motu æquabili; ſequitur, quòd 6 horarum ſpatio per Meridianum tranſeat

quadrans unus tum æquatoris, tum cujuſque circuli paralleli; unde differentia aſcenſionalis in tempus converſa, & addita vel ſubtracta (prout Solis locus exigit) 6 horis, dabit tempus correſpondens arcui ſemidiurno.

Differentiam verò aſcenſionalem, vel quemcunque æquatoris arcum in tempus convertere, non eſt aliud quàm determinare, quanto tempore arcus ille per Meridianum tranſeat. Cùm igitur horis ſingulis una vigeſima quarta æquatoris pars ſeu 15 gradus per Meridianum tranſeant; invenietur tempus cuicunque arcui correſpondens, dividendo ejus gradus & minuta per 15 : vel, faciliùs, ea multiplicando per 4, mutatis in producto gradibus in minuta, & minutis primis in ſecunda. Sic aſcenſionalis differentia 26 gr. 41′ multiplicata per 4 dat, ſignis mutatis, 1 h. 46′. 44″. ſeu Hor. 1. 46′. 44″.

G.	
26	41′
	4
106′	44″ *ideſt*
1 h	46 44

Operationis ratio obvia eſt; nam Gradus deprimere ad Minuta, & prima mutare in ſecunda, eſt productum dividere per 60. Multiplicare verò per 4, & productum dividere per 60, non eſt aliud quàm dividere per 15, ut requiritur.

Tempus differentiæ aſcenſionali correſpondens viz: 1 h 46′. 44″. (quia Gemini ſunt ſignum boreale) additum 6 horis, dat 7 h 46′. 44″. quibus arcus CÆ (cui ſimilis eſt BO) totus Meridianum tranſit, ſeu tempus ſemidiurnum. Unde Sol in initio Geminorum oritur Leodii, horis 7. 46′. 44″ ante meridiem, id eſt, horâ 4. 13′. 16″; occidit autem horâ 7. 46′. 44″.

PROBLEMA IV.

Datâ Poli altitudine cum declinatione Solis, vel differentiâ aſcenſionali, invenire amplitudinem ortivam vel occiduam.

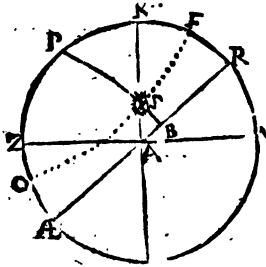
Poſitis omnibus ut in *Probl.* 3. In triangulo ABC, angulus C eſt rectus, angulus A 39 gr. 20′. & latus BC 20 gr. 12′. Ex his (*ut in* 7. *caſu* 1. *ſchem.*) invenitur amplitudo ortiva vel occidua AB 33 gr. 1′.

Similiter datâ in eodem triangulo differentiâ aſcenſionali AC; ex AC & angulis A, C, invenitur (*ut in* 8. *caſu* 1. *ſchem.*) baſis AB.

Et è converſo, notâ baſi AB ſeu amplitudine ortivâ cum angulis A, C, invenitur (*ut in* 1. *caſu* 1. *ſchem.*) declinatio Solis BC, & (*ut in* 2. *caſu*) differentia aſcenſionalis AC.

PROBLEMA V.

Cognitâ altitudine Poli, & declinatione Solis, determinare tempus quo, Sol perveniet ad Circulum veri Orientis vel Occidentis, ſeu ad verticalem primarium: & è converſo.



Sit PÆR Meridianus. ZN Horizon. ÆR Æquator, eique parallelus OF. K Zenith. PS circulus declinationis & ſimul horarius tranſiens per centrum Solis exiſtentis in S puncto verticalis primarii KSA. His poſitis, in triangulo ABS, angulus B (*per* 6. *c.* 1. *h*) eſt rectus anguli SAB menſura eſt KR diſtantia Zenith ab æquatore, quæ æqualis eſt altitudini poli; adeoque Leodii angulus SAB eſt 50 gr. 40′. Arcus SB eſt declinatio Solis, quæ in initio Geminorum eſt 20 gr. 12′. Ex his (*ut in* 5. *caſu* 1. *ſchem.*) invenitur arcus AB 17 gr. 33′. Unde ejus complementum BR diſtantia Æquatoria Solis à Meridiano PKR, eſt gr. 72. 27′. quæ ad tempus reducta (*ut in Corol. Probl.* 3.) dat 4. horas. 49′. 48″. Sol igitur in initio Geminorum attingit circulum veri Orientis horâ 7. 10′. 12″. ante, & veri Occidentis horâ 4. 49′. 48″ poſt meridiem.

E converſo. Si detur hora quâ Sol appellit ad verticalem primarium KSA, ejus diſtantia à meridie converſa (*ut in Corol. Probl.* 3.) in gradus æquatoris dabit arcum BR, adeoque & ejus complementum BA. Ex quo & angulis B, BAS, invenitur (*ut in* 4. *caſu* 1. *ſchem.*) SB declinatio Solis.

PROBLEMA VI.

Cognitâ Solis declinatione & Poli altitudine, altitudinem Solis quâvis diei horâ invenire, v. g. *horâ octavâ.*

Fiat eadem conſtructio, quæ *in Probl.* 5. In triangulo

PKS arcus PK est complementum altitudinis Poli, adeoque gr 39. 20′. Arcus PS est complementum declinationis solaris SB, quæ si ponamus Solem in initio Tauri *v. g.* est 11 gr. 30′ unde PS habebit 78 gr. 30′. Angulus verò SPK (*per* 3. *c.* 1. *h*) æqualis est arcui æquatoris BR, qui mensurat distantiam æquatoriam Solis à Meridiano PÆR; & invenitur dicendo, ut 1 hora ad 15 gradus: sic tempus inter horam constituram & meridiem, viz: 4 horæ, ad aliud. Prodeunt 60 gr, adeoque tantus est angulus SPK. Ex hoc angulo & lateribus PK 39 gr. 20′, PS 78 gr. 30′. invenitur (*ut in* 2. *casu* 2. *schem.*) latus KS gr 62. 19′. cujus complementum SA 27 gr. 41′. est altitudo Solis quæsita.

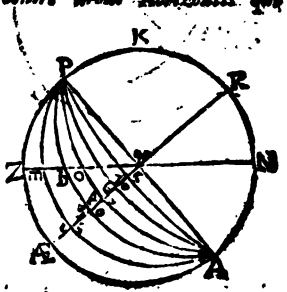
PROBLEMA VII.

Cognitâ Poli altitudine cum Solis declinatione & altitudine, invenire horam diei vel Solis Azimuth.

Detur altitudo Solis (*vide Fig. Probl.* 5.) SA *v. g.* 27. gr. 41′. erit ejus complementum KS 62. gr. 19′. Detur etiam declinatio Solis SB, *v. g.* 11 gr. 30′. erit ejus complementum PS 78. gr. 30′. Sit altitudo Poli ZP 50 gr. 40′. erit ejus complementum PK 39 gr. 20′. Ergo in triangulo PSK nota sunt omnia latera. Ex quibus (*ut in Prop.* 4. *c.* 3. *h.*) invenitur angulus P 60′ gr. qui in tempus mutati dant 4 horas. Ex iisdem inveniri potest angulus PKS, id est (*per* 3. *c.* 1. *h*) arcus AZ Solis Azimuth.

PROBLEMA VIII.

Cognitâ Poli altitudine invenire arcus Horizontis qui inter Meridianum & alios Horarios circulos intercipiuntur.



Sit PÆR Meridianus. ÆR Æquator. ZN Horizon. P Polus. K Zenith. PZA, PMA, PBA &c. circuli Horarii. In triangulis ZPM, ZPB &c. quia Meridianus ZP transit per Zenith K polum Horizontis, angulus Z (*per* 6. *c.* 1. *h*) est rectus. Anguli verò ad P noti

sunt, nam (*per 6. c. 1. h*) sunt ejusdem mensuræ cum arcubus æquatoriis Æ 15. Æ 30. &c. Denique latus ZP est altitudo Poli quæ supponitur nota. Ex his tribus (*ut in 4. casu 1. schem.*) inveniuntur arcus ZM, ZB, ZO &c. quæsiti.

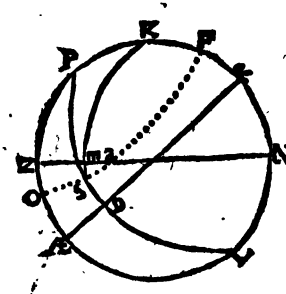
PROBLEMA IX.

Datâ Poli altitudine & declinatione Solis, invenire durationem Crepusculi.

Docent Astronomi plerique Crepusculum manè inchoari, & vesperi desinere, quando Sol deprimitur 18 gradibus infra Horizontem. Quærimus quantitatem temporis interjecti inter initium Crepusculi matutini & ortum Solis, sive inter ejusdem occasum & finem Crepusculi vespertini.

Sit PRÆ Meridianus. ZN Horizon. K Zenith. Circulus verticalis KMS. Polus P. Circulus declinationis & simul Horarius PSL. Æquator ÆR, eique parallelus circulus OF. Solis locus S in initio Geminorum.

In triangulo PSK latus PK est complementum altitudinis Poli, adeoque hîc Leodii est 39. gr. 20′. Latus PS habetur subtrahendo ex quadrante PD borealem Solis declinationem DS, quæcùm sit in initio Geminorum 20. gr. 12′. latus PS est 69. gr. 48′. Latus KS habetur addendo Solis profunditatem SM, seu 18 gr. quadranti KM; adeoque KS est 108 gr. Notis sic tribus lateribus PK, PS, KS invenitur (*per 4. c. 3. h*) angulus P, id est (*per 3. c. 1. h*) arcus RD seu FS 165 gr. 34′. Hinc subtrahe arcum semidiurnum FA, qui (*ex dictis*) tunc est 116 gr. 41′. & residuum SA 48 gr. 53′. intempus conversum dabit 3 horas. 15′. 32″. pro Crepusculo quæsito.

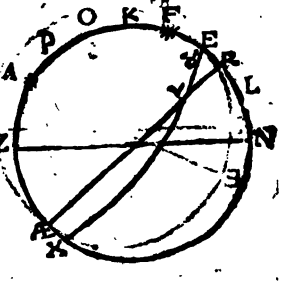


Nota quòd, quando Sol est in signis Australibus, ejus declinatio addenda sit quadranti PD.

PROBLEMA X.

Alicujus ſtellæ fixæ declinationem, aſcenſionem rectam & longitudinem & latitudinem inveſtigare.

1.o Stellæ propoſitæ ad Meridianum ZKN appulſæ capiatur altitudo quadrante exquiſito. Si ſtella ſit in quadrante boreali KZ, & infra Polum P, *v. g.* in A; altitudini inventæ AZ addatur arcus ZÆ profunditas æquatoris ÆR infra Horizontem ZN, & notus erit arcus AÆ declinatio quæſita. Si ſit inter Polum P & Zenith K, *v. g.* in O, ejus à Zenith diſtantia OK addita Poli altitudini PZ ſeu KR, dabit OR declinationem quæſitam. Si ſtella ſit in quadrante Auſtrali, eandemque habeat altitudinem Meridianam cum æquatore, nullam habet declinationem. Si verò altior ſit, *v. g.* in F; ejus altitudini FN ſubtracta æquatoris altitudo RN relinquet declinationem FR quæſitam. Si denique ſtella ſit inter æquatorem & Horizontem, *v. g.* in L, ejus declinatio LR eſt Auſtralis, & habetur ſubtrahendo ſtellæ altitudinem LN. Ex æquatoris altitudine RN.

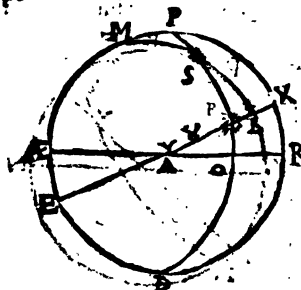


2.o Ut inveniatur aſcenſio recta alicujus ſtellæ, menſuretur accuratè (numerando Penduli vibrationes, vel aliter) tempus inter Meridiem proximè præcedentem, & ſtellæ appulſum ad Meridianum, qui diligenter debet obſervari. Tempus verò inventum mutetur in gradus æquatoris (quod fit temporis inventi horas & minuta multiplicando per 15) quibus addatur aſcenſio Solis recta pro Meridie præcedenti (*per Probl. 3. b*) computata. Eritque eorum ſumma aſcenſio ſtellæ recta, quæ quæritur.

Ex. gr. Sole exiſtente in initio Geminorum, adeoque habente (*per Probl. 2.*) aſcenſionem rectam 57 gr. 48′; obſervatus fuerit anno 1704. noctis horâ 10. 10′. 59″. 56‴. appulſus ſtellæ, *v. g.* Arcturi ad Meridianum. Tempus hoc converſum in æquatoris gradus, dat 152 gr. 44′. 59″. 2‴.

quibus ſi addatur Solis aſcenſio recta; ſumma 210 gr. 32′. 59″. erit Arcturi aſcenſio recta quæſita.

3.o Ut exploretur ſtellæ alicujus longitudo vel latitudo, ſit ÆR Æquator. EX Ecliptica. ÆPRD Colurus Soſtitialis. PSD circulus declinationis ſtellæ exiſtentis in S, cujus proinde declinatio eſt SO. Demum ſit MSL circulus latitudinis ejuſdem ſtellæ. His poſitis, erunt in triangulo MPS, latus PS complementum declinationis SO, quæ ſupponitur nota; latus MP 23. gr. 30′. diſtantia inter Polos æquatoris & Eclipticæ; angulus verò MPS (*per* 3. *c.* 1. *h*) menſuratur ab æquatoris arcu ÆO, qui habetur addendo vel ſubtrahendo quadranti ſtellæ



propoſitæ aſcenſionem rectam, vel ejus exceſſum ſupra ſemicirculum. Quid autem horum fieri oporteat, non aliunde melius quàm ex ipſa ſphæra conſtabit, in qua attentè conſiderari debet circulorum propoſitorum ſitus, & cum ſiderum cælorumque motu conferri. Ex his tribus (*ut in* 7. *caſu* 2. *vel* 3. *ſchem.*) invenitur latus MS, complementum arcûs SL latitudinis quæſitæ: & (*ut in* 8. *caſu* 2. *vel* 3. *ſchem.*) angulus SMP, quem (*per* 3. *c.* 1. *h*) menſurat arcus XL, qui (prout ſtellæ locus exigit) additus aut ſubtractus uni pluribuſve circuli quadrantibus, dabit longitudinem quæſitam.

Nota 2.o ſtellas fixas circumagi motu proprio circa Polos Eclipticæ, & annis ſingulis conficere 50 circiter ſecunda. Unde fit ut, quamvis earum latitudines ſint ſemper eædem, mutentur tamen longitudines, declinationes, & aſcenſiones rectæ; quia mutatur earum ſitus reſpectu primi Mobilis & æquatoris. Stellarum aliquot nobiliorum longitudines, &c. exhibet Tabula ſequens, à Ricciolo computatas pro anno 1700 completo, cum aſcenſionum & declinationum differentiis pro annis 100, quarum ope cuivis anno poſſunt facilè accommodari.

TABULA

TABULA

Longitudinum, Latitudinum, Ascensionum Rect. & Declinationum Stellarum cum Differentiis Ascension. & Declinat.

	Lucida Lyræ	Arcturus	Cor Leonis
	Gr.	Gr.	Gr.
Longitudo	281. 06'. 17"	200. 04'. 27"	145. 38'. 32"
Latitudo	61. 47. 00	31. 00. 40	0. 26. 29
Ascensio recta	276. 39. 32	210. 33. 02	148. 04. 15
Differ. Ascen.	0. 50. 00	1. 11. 00	1. 32. 39
Declinatio	38. 32. 16	20. 48. 02"	13. 35. 16
Differ. Declin.	A. 0. 04. 00	S. 0. 29. 30	S. 0. 28. 30
	Canicula	Capella	Caput Andr.
	Gr.	Gr.	Gr.
Longitudo	111. 40'. 27"	77. 40'. 02".	10. 09'. 03"
Latitudo	15. 57. 10	23. 51. 45	25. 42. 10
Ascensio recta	110. 54. 33	73. 35. 56	358. 14. 08
Differ. Ascen.	1. 20 00	1. 49. 00	1. 17. 00
Declinatio	5. 59. 12	45. 40. 00	27. 27. 26
Differ. Declin.	S. 0. 12. 00.	A. 0. 10 00	A. 0. 34. 00

Ad Tabulæ hujus usum est diligenter observandum

1.° Quòd Fixarum longitudo augeatur annis singulis min. 50". 40‴. quæ si multiplices per numerum annorum interjectum inter an. 1700 & tempus datum, productumque addas longitudini in Tabula inventæ; summa erit Fixæ longitudo ad tempus datum.

2.° Latitudines Fixarum nunquam mutantur.

3.° Si quæratur Stellæ alicujus Ascensio recta, fiat ut anni 100, ad differentiam Ascensionis in Tabula positam: sic anni inter an. 1700 & tempus datum, ad aliud. Quartus terminus additus Ascensioni in Tabula inventæ, dabit Ascensionem quæsitam. Eadem methodus tenenda est, quando quæritur declinatio. Quartus tamen terminus addi debet, vel subtrahi declinationi, quæ in Tabula habetur; prout vel litera A addendum, vel litera S subtrahendum indicat.

Nota 1.° Quòd si tempus, pro quo quæritur longitudo, vel

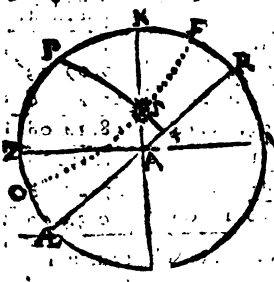
aſcenſio, vel declinatio ſtellæ, anteceſſerit annum 1700; adhibenda ſit ſubtractio, ubi additione uſi ſumus, & è contrà.

Nota 2.o Boream verſùs declinare ſtellas omnes in hac Tabula contentas: Caniculæ tamen latitudo eſt Auſtralis, licèt declinatio ſit Borealis.

PROBLEMA XI.

Ex dictis invenire noctis horam per ſtellas.

Sit conſtructio eadem quæ *Probl.* 5. & 6. ſed S ſit locus ſtellæ, Arcturi v. g. cujus inventa ſit altitudo AS 50 gr. adeoque ejus complementum KS gr. 40. Deinde, quia ZP Poli altitudo eſt 50 gr. 40'; erit ejus complementum PK 39 gr. 20'. Denique latus PS habetur ſubtrahendo SB ſtellæ declinationem (vel eam addendo, ſi ſit Auſtralis) ex quadrante PB. Cùm igitur Arcturi declinatio ſit Borealis, & (anno 1704) 20. gr. 46'. 51''. erit PS 69 gr. 13'. 9''. Ex his tribus arcubus invenitur (*per* 4. *c.* 3. *h*) in triangulo PSK angulus P, adeoque (*per* 3. *c.* 1. *h*) arcus BR 33 gr. 56'. Hoc facto, ex Arcturi aſcenſione rectâ 210 gr. 33'; ſubtrahatur (ſi minor ſit) Solis, exiſtentis *ex. gr.* in initio Cancri, aſcenſio recta 90 gr. Reſiduum 120 gr. 33'. erit diſtantia Æquatoria Solis ab Arcturo; cui diſtantiæ ſi addatur BR 33 gr. 56' habebitur Solis à Meridiano diſtantia Æquatoria 154 gr. 29'. quæ in tempus converſa dat 10 h. 17'. 56''. noctis horam quæſitam.



Nota 1.o Quòd ſi aſcenſio recta Solis major ſit aſcenſione rectâ ſtellæ, hæc debeat ex illa ſubtrahi, ut habeatur diſtantia Solis Æquatoria à ſtella propoſita.

Nota 2.o Quòd ſi BR diſtantia ſtellæ à Meridiano ſit (ut in hac Figura) Occidentalis, addi debeat arcus BR Solis diſtantiæ Æquatoriæ ab ipſa ſtella: ſin Orientalis ſit, debet ſubtrahi.

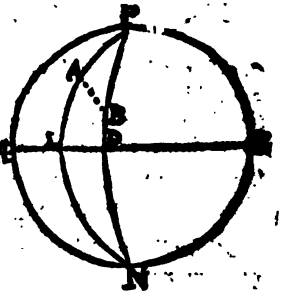
Ut ad manum ſemper haberentur declinationes & aſcen-

tiones ſtellarum, quas utiliſſimi hujus Problematis ſolutio requirit; in Tabula præcedente ſelectæ ſunt ſtellæ in Hæmiſphærio noſtro conſpicuæ & ita diſperſæ, ut earum aliquis ſemper inſervire poſſit ad finem intentum.

PROBLEMA XII.

Datis Longitudine & Latitudine duarum Urbium vel Stellarum, invenire earum diſtantiam.

Sit primus Meridianus ÆPR. Æquator ÆR. Sit etiam A Urbs aliqua, Leodium *v. g.* ejuſque Meridianus PLN, Longitudo ÆL 28 gr. 38′. Latitudo AL 50 gr. 40′. Sit B Hieroſolyma, ejuſque Meridianus PDN, Longitudo ÆD 63 gr. 37′. Latitudo DB 32 gr. Hinc in triangulo APB cognoſcuntur latera AP, BP complementa latitudinum notarum AL, DB; unde AP eſt 39 gr. 20′. BP 58 gr. Angulus verò ABP menſuratur (*per* 3. *c.* 1. *b*) ab arcu LD longitudinum datarum differentiâ, quæ eſt 34 gr. 59′. Ex his tribus invenitur (*ut in* 1. *caſu* 2. *vel* 3. *ſchem.*) arcus AB 31 gr. 46′. Qui (methodo prædictâ) in Milliaria converſus, dat Milliaria Bononienſia 2047 & paſſus 816, ſeu Milliaria Horaria 682 & paſſus 1816.



Si A & B intelligantur eſſe duæ ſtellæ, & ÆR Ecliptica; eodem modo invenietur earum diſtantia.

Idem faciliùs præſtatur ope ſphæræ, ex qua ſi diſtantia inter loca propoſita circino excipiatur, & applicetur alicui circulo maximo in gradus diviſo, ſtatim innoteſcet quantus ſit arcus interceptus.

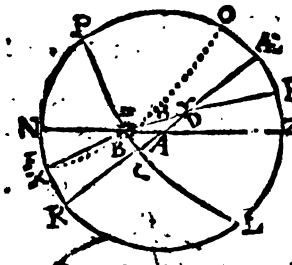
PROBLEMA XIII.

Cognitâ ſtellæ aſcenſione rectâ & declinatione, invenire ejus aſcenſionem obliquam, amplitudinem orituam vel occiduam, & arcum diurnum vel nocturnum.

Sit PRÆ Meridianus. ÆR Æquator. B locus ſtellæ in Horizonte obliquo NZ. Horizon rectus & circulus declinationis

ſtellæ ſit PCL. Erit ſtellæ aſcenſio recta DC; obliqua AD; differentia aſcenſionalis CA; amplitudo ortiva BA; ſtellæ declinatio BC. His poſitis,

In triangulo BAC, declinatio BC ſupponitur nota; angulus C eſt (*per 6. c. 1. h*) rectus; & angulus BAC menſuratur ab arcu NR complemento Poli altitudinis NP. Ex his tribus invenitur 1.o (*ut in 2. caſu 1. ſchem.*) differentia aſcenſionalis CA, quæ (ſi ſtella Borealis ſit) ſubtracta aſcenſioni rectæ CD, relinquit notam AD aſcenſionem obliquam. 2.o Ex iiſdem (*ut in 7. caſu 1. ſchem.*) exploratur BA amplitudo ortiva vel occidua. 3.o Addendo (ſi ſtella ad Septentrionem declinet) CA, aſcenſionalem differentiam Æquatoris, quadranti AÆ, habetur CÆ, eique ſimilis BO arcus ſemidiurnus, adeoque & arcus diurnus; qui ſubtractus gradib. 360 relinquit arcum nocturnum notum.



PROBLEMA XIV.

Altitudinem Poli invenire per duas ſtellas ſimul orientes, quarum notæ ſint declinationes & aſcenſiones rectæ.

Sit Meridianus ZKN, Æquator ÆR. Polus P. Horizon ZN, in quo ſint ſtellarum ſimul orientium loca M, B. Circuli declinationum PMA. PBA. Aſcenſionum rectarũ differentia IO, quæ notis ipſis aſcenſionib. nota erit.

Hinc in triangulo MPB cognoſcuntur latera PM, PB, complementa datarum declinationum IM, OB; angulus MPB menſuratur (*per 3. c. 1. h*) ab arcu noto IO. Ex his (*ut in 7. caſu 2. vel 3. ſchem.*) invenitur angulus PBM, ſeu PBZ. Dein in triangulo ZPB, ex angulo Z (*per 6. c. 1. h*) recto, angulo PBZ & latere PB notis, invenitur (*ut in 1. caſu 1. ſchem.*) latus ZP Poli P altitudo quæſita.

CANONES SINUUM ET TANGENTIUM PRO DECIMO QUOQUE MINUTO CUM EORUM LOGARITHMIS.

OPeri præstituta brevitas integros Sinuum, Tangentium, & Secantium Canones prohibebat inseri; & ex alia parte videbatur discentium exercitationi non satis prospici, si prorsùs omitterentur. Selegi proinde Sinus & Tangentes pro decimo quoque minuto, cum eorum Logarithmis; Secantibus, quarum vix est usus, penitùs prætermissis. Si quis tamen Sinus & Tangentes intermedios requirat; sic facilè investigabit.

Datis Sinubus vel Tangentibus Minutorum decimalium, invenire quoscunque intermedios.

Quæratur *v. g.* Sinus arcûs dati anguli 43. gr. 14′.

1.o Desumantur ex tabulis Sinus proximè major (43. gr. 20′. qui est 6862416) & proximè minor (43. gr. 10′. qui est 6841229) & inquiratur eorum differentia (21187.)

2.o Fiat, ut 10 ad excessum arcûs dati (43. gr. 14′.) supra arcum proximè minorem (43. gr. 10′.) id est, ut 10 ad 4: ita 21187 differentia Sinuum ex tabula desumptorum ad aliud.

Quartus terminus $8474\frac{8}{10}$ seu 8475 additus Sinui proximè minori (6841229) dabit (6849704) Sinum anguli dati, vel ipsissimum qui in tabulis reperitur, vel ab illo vix differentem.

Eadem operatio institui potest ad Logarithmicos Sinus inquirendos, licèt non ita exactè, sufficienter tamen ad præsens institutum.

Quod dictum de Sinubus, de Tangentibus etiam est intelligendum. Hinc etiam & ex *Prop.* 3. *c* 1. *P.* 1. colliges quomodò ex Sinu vel Tangente datâ inveniantur minuta correspondentia.

Grad.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
		9	0	0	9
0	0				
	10	290.89	290.89	7.4637255	7.4637273
	20	581.77	581.78	7.7647537	7.7647610
	30	872.65	872.69	7.9408419	7.9408584
	40	1163.53	1163.61	8.0657769	8.0658057
	50	1454.39	1454.54	8.1626808	8.1627267
1	0	1745.24	1745.51	8.2418553	8.2419215
	10	2036.08	2036.50	8.3087941	8.3088842
	20	2326.90	2327.53	8.3667769	8.3668945
	30	2617.69	2618.59	8.4179190	8.4180679
	40	2908.47	2909.70	8.4636649	8.4638486
	50	3199.22	3200.86	8.5050447	8.5052671
2	0	3489.95	3492.08	8.5428192	8.5430838
	10	3780.65	3783.35	8.5775660	8.5778766
	20	4071.31	4074.69	8.6097341	8.6100943
	30	4361.94	4366.09	8.6396796	8.6400931
	40	4652.53	4657.57	8.6676893	8.6681598
	50	4943.08	4949.13	8.6939980	8.6945292
3	0	5233.60	5240.78	8.7188002	8.7193958
	10	5524.06	5532.51	8.7422586	8.7429222
	20	5814.48	5824.34	8.7645111	8.7652465
	30	6104.85	6116.26	8.7856753	8.7864861
	40	6395.17	6408.29	8.8058523	8.8067422
	50	6685.44	6700.43	8.8251299	8.8261026
4	0	6975.65	6992.68	8.8435845	8.8446437
	10	7265.80	7285.05	8.8612833	8.8624327
	20	7555.89	7577.55	8.8782854	8.8795286
	30	7845.91	7870.17	8.8946433	8.8959842
	40	8135.87	8162.91	8.9104039	8.9118460
	50	8425.76	8455.81	8.9256089	8.9271560
5	0	8715.57	8748.87	8.9402960	8.9419518
	10	9005.32	9042.06	8.9544991	8.9562672
	20	9294.99	9335.40	8.9682487	8.9701330
	30	9584.58	9628.90	8.9815729	8.9835769
	40	9874.08	9922.57	8.9944968	8.9966243
	50	10163.51	10216.41	9.0070436	9.0092984

Gr.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
89	60	100000.00	Infinit.	10.0000000	Infin. t.
	50	99999.58	34377371.00	9.9999982	12.5362727
	40	99998.30	17188540.00	9.9999927	12.2352390
	30	99996.19	11458865.00	9.9999835	12.0591416
	20	99993.23	8593979.16	9.9999706	11.9341943
	10	99989.42	6875008.76	9.9999541	11.8372733
88	60	99984.77	5728996.16	9.9999338	11.7580785
	50	99979.27	4910388.06	9.9999100	11.6911158
	40	99972.92	4296407.73	9.9998824	11.6331055
	30	99965.73	3818845.93	9.9998512	11.5819321
	20	99957.69	3436777.09	9.9998162	11.5361514
	10	99948.81	3124157.67	9.9997776	11.4947329
87	60	99939.08	2863625.33	9.9997354	11.4569162
	50	99928.51	2643159.96	9.9996894	11.4221234
	40	99917.09	2454175.78	9.9996398	11.3899057
	30	99904.82	2290376.55	9.9995869	11.3599059
	20	99891.71	2147040.10	9.9995297	11.3318402
	10	99877.75	2020555.35	9.9994688	11.3054708
86	60	99861.95	1908113.67	9.9994044	11.2806042
	50	99847.31	1807497.74	9.9993364	11.2570778
	40	99830.81	1716933.69	9.9992646	11.2347535
	30	99813.48	1634985.55	9.9991892	11.2135139
	20	99795.29	1560478.41	9.9991101	11.1932578
	10	99776.27	1492441.70	9.9990273	11.1738074
85	60	99756.40	1430066.63	9.9989408	11.1553563
	50	99735.69	1372673.79	9.9988506	11.1375673
	40	99714.13	1319688.30	9.9987567	11.1204714
	30	99691.73	1270620.47	9.9986591	11.1040158
	20	99668.49	1225050.55	9.9985579	11.0881540
	10	99644.40	1182616.67	9.9984529	11.0728440
84	60	99619.47	1143005.23	9.9983442	11.0580482
	50	99593.69	1105943.10	9.9982318	11.0437328
	40	99567.08	1071191.26	9.9981158	11.0298670
	30	99539.62	1038539.71	9.9979960	11.0164231
	20	99511.32	1007803.11	9.9978725	11.0033757
	10	99482.17	978817.32	9.9977453	10.9907016

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
6	0	10452.85	10510.42	9.019 2346	9.0216202
	10	10742.10	10804.62	9.0310890	9.0336093
	20	11031.26	11098.99	9.0426249	9.0452836
	30	11320.32	11393.56	9.0538588	9.0566595
	40	11609.29	11688.31	9.0648057	9.0677522
	50	11898.16	11983.28	9.0754799	9.0785760
7	0	12186.93	12278.46	9.0858945	9.0891438
	10	12475 60	12573 84	9.0960615	9.0994678
	20	12764.16	12869 43	9.1059924	9.1095594
	30	13052.62	13165.25	9.1156977	9.1194291
	40	13340.96	13461.29	9.1251872	9.1290868
	50	13629.19	13757.57	9.1344702	9.1385417
8	0	13917.31	14054.08	9.1435553	9.1478025
	10	14205.31	14350.84	9.1524507	9.1568773
	20	14493.19	14647 84	9.1611639	9.1657737
	30	14780.94	14945 10	9.1697021	9.1744988
	40	15068 57	15242.61	9.1780721	9.1830595
	50	15356.07	15540.40	9.1862802	9.1914621
9	0	15643.45	15838 44	9.1943324	9.1997125
	10	15930.69	16136.77	9.2022345	9.2078165
	20	16217.79	16435.37	9.2099917	9.2157795
	30	16504.76	16734.26	9.2176092	9.2236065
	40	16791.59	17033 44	9.2250918	9.2313024
	50	17078.28	17332.92	9.2324440	9.2388717
10	0	17364.82	17632.70	9.2396702	9.2463188
	10	17651.21	17932 78	9.2467746	9.2536477
	20	17937.46	18233.18	9.2537609	9.2608625
	30	18223.55	18533.90	9.2606330	9.2679669
	40	18509.44	18834.95	9 2673945	9.2749644
	50	18795.27	19136 32	9.2740487	9.2818585
11	0	19080.90	19438.03	9.2805988	9.2886523
	10	19366.36	19740.08	9.2870480	9.2953489
	20	19651.66	20042.48	9.2933993	9.3019514
	30	19936.79	20345.23	9.2996553	9.3084626
	40	20221.76	20648.34	9.3058189	9.3148851
	50	20506.55	20951.81	9.3118926	9.3212216

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
83	60	99452.18	9511436.45	9.9976143.	10.9783798
	50	99421.36	9255530.35	9.9974797	10.9663907
	40	99389.69	9009826.61	9.9973414	10.9547164
	30	99357 18	8776688.74	9.9971993	10.9433405
	20	99323.83	8555554.68	9.9970535	10.9322478
	10	99289.64	8344955 57	9.9969040	10.9214240
82	60	99254.62	8144344.64	9 9967507	10.9108562
	50	99218.74	7953022 4	9.9965937	10.9005322
	40	99182.03	7770355.06	9.9964330	10.8904406
	30	99144.49	7595755.41	9.9962686	10.8805709
	20	99106.09	7428706.64	9 9961004	10.8709132
	10	99066 87	7268725.55	9.9959284	10.8614583
81	60	99026 80	7115369.97	9 9957528	10.8521975
	50	98985.90	6968233.35	9 9955734	10 8431227
	40	98944.16	6826943 7	9.9953902	10.8342263
	30	98901.58	6991156.2	9.9952033	10.8255012
	20	98858.17	6560555.38	9 9950126	10 8169405
	10	98813.92	6434842.8	9.9948181	10.8085379
80	60	98768.83	6313755.15	9.9946199	10 8002875
	50	98722.91	6197027.9	9.9944180	10.7921835
	40	98676.15	6084438.81	9 9942122	10.7842205
	30	98628.56	5975764.4	9 9940027	10.7763935
	20	98580.13	5870804.2	9 9937894	10.7686976
	10	98530.87	5769368 8	9.9935723	10.7611283
79	60	98480.77	5671281 8	9.9933515	10.7536812
	50	98429.85	5576378.6	9.9931269	10.7463523
	40	98378.08	5484505 2	9.9928984	10 7391375
	30	98325.49	5395511.72	9.9926661	10.7320331
	20	98272.06	5309279 3	9.9924301	10.7250356
	10	98217 81	5225664.7	9.9921902	10 7181415
78	60	98162 71	5144554.0	9 9919466	10.7133477
	50	98106.80	5065832	9.9916991	10 7046511
	40	98050.05	4989402.7	9.9914478	10.6980486
	30	97993.47	4915157.0	9.9911927	10.6915374
	20	97934.06	4843004.5	9.9909338	10 6851149
	10	97874.83	4772857	9.9906710	10.6787784

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
12	0	20791.17	21255.65	9.3178789	9.3274745
	10	21075.61	21559.88	9.3237802	9.3336463
	20	21359.88	21864.48	9.3295988	9.3397391
	30	21643.96	22169.47	9.3353368	9.3457552
	40	21927.86	22474.85	9.3409963	9.3516968
	50	22211.58	22780.63	9.3465794	9.3575658
13	0	22495.11	23086.82	9.3520880	9.3633641
	10	22778.44	23393.42	9.3575240	9.3690937
	20	23061.59	23700.44	9.3628892	9.3747563
	30	23344.54	24007.87	9.3681853	9.3803537
	40	23627.29	24315.75	9.3734139	9.3858876
	50	23909.84	24624.05	9.3785767	9.3913595
14	0	24192.19	24932.80	9.3836752	9.3967711
	10	24474.33	25242.00	9.3887109	9.4021237
	20	24756.27	25551.65	9.3936852	9.4074189
	30	25038.00	25861.76	9.3985996	9.4126581
	40	25319.52	26172.34	9.4034554	9.4178425
	50	25600.82	26483.39	9.4082539	9.4229735
15	0	25881.90	26794.92	9.4129962	9.4280525
	10	26162.77	27106.93	9.4176837	9.4330804
	20	26443.42	27419.44	9.4223176	9.4380587
	30	26723.84	27732.45	9.4268988	9.4429883
	40	27004.03	28045.97	9.4314286	9.4478704
	50	27284.00	28359.99	9.4359080	9.4527061
16	0	27563.74	28674.54	9.4403381	9.4574964
	10	27843.24	28989.61	9.4447197	9.4622423
	20	28122.51	29305.21	9.4490540	9.4669448
	30	28401.53	29621.35	9.4533418	9.4716048
	40	28680.32	29938.03	9.4575840	9.4762233
	50	28958.87	30255.27	9.4617816	9.4808011
17	0	29237.17	30573.07	9.4659353	9.4853390
	10	29515.22	30891.43	9.4700461	9.4898380
	20	29793.03	31210.36	9.4741146	9.4942988
	30	30070.58	31529.88	9.4781418	9.4987223
	40	30347.88	31849.98	9.4821285	9.5031092
	50	30624.92	32170.67	9.4860749	9.5074602

G.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
77	60	97814.76	470463.01	9.9904044	10.6725255
	50	97753.86	463824.57	9.9901339	10.6663537
	40	97692.15	457362.87	9.9898597	10.6602609
	30	97629.60	451070.85	9.9895815	10.6542448
	20	97566.23	444941.81	9.9892995	10.6483032
	10	97502.03	438969.40	9.9890137	10.6424342
76	60	97437.01	433147.59	9.9887239	10.6366359
	50	97371.16	427470.66	9.9884303	10.6309063
	40	97304.48	421933.18	9.9881329	10.6252437
	30	97236.99	416529.98	9.9878315	10.6196463
	20	97168.67	411256.14	9.9875263	10.6141124
	10	97099.54	406107.00	9.9872171	10.6086405
75	60	97029.57	401078.09	9.9869041	10.6032289
	50	96958.79	396165.18	9.9865872	10.5978763
	40	96887.18	391364.20	9.9862663	10.5925811
	30	96814.76	386671.31	9.9859416	10.5873419
	20	96741.52	382082.81	9.9856129	10.5821575
	10	96667.46	377595.19	9.9852803	10.5770265
74	60	96592.58	373205.08	9.9849438	10.5719475
	50	96516.88	368909.27	9.9846033	10.5669196
	40	96440.37	364704.67	9.9842589	10.5619413
	30	96363.05	360588.35	9.9839105	10.5570117
	20	96284.90	356557.49	9.9835582	10.5521296
	10	96205.94	352609.38	9.9832018	10.5472939
73	60	96126.17	348741.44	9.9828416	10.5425036
	50	96045.58	344951.20	9.9824774	10.5377577
	40	95964.18	341236.26	9.9821092	10.5330552
	30	95881.97	337594.34	9.9817370	10.5283952
	20	95798.95	334023.26	9.9813608	10.5237767
	10	95715.12	330520.91	9.9809805	10.5191989
72	60	95630.48	327085.26	9.9805963	10.5146610
	50	95545.02	323714.38	9.9802081	10.5101620
	40	95458.76	320406.38	9.9798158	10.5057012
	30	95371.69	317159.48	9.9794195	10.5012777
	20	95283.82	313971.94	9.9790192	10.4968908
	10	95195.14	310842.10	9.9786148	10.4925398

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
18	0	30901.70	32491.97	9.4899824	9.5117760
	10	31178.22	32813.87	9.4938513	9.5160575
	20	31454 48	33136.39	9.4976824	9.5203052
	30	31730.47	33459.58	9.5014764	9.5245199
	40	32006.19	33783.30	9.5052339	9 5287021
	50	32281.64	34107.71	9.5089556	9.5328526
19	0	32556 82	34432.76	9.5126419	9.5369718
	10	32831.72	34758 46	9.5162936	9.5410606
	20	33106.34	35084.83	9.5199112	9.5451193
	30	33380 69	35411.86	9.5234953	9 5491487
	40	33654 75	35739.56	9.5270463	9.5531492
	50	33928.53	36067.95	9.5305650	9.5571214
20	0	34201.02	36397.02	9.5340517	9.5610658
	10	34475.22	36726.80	9.5375069	9.5649831
	20	34748.13	37057.28	9 5409314	9.5688735
	30	35020.74	37388.47	9.5443253	9.5727377
	40	35293.06	37720.38	9.5476893	9.5765761
	50	35565.08	38053 03	9.5510237	9.5803892
21	0	35836.79	38386.40	9.5543292	9 5841774
	10	36108.21	38720.53	9.5576060	9.5879413
	20	36379.32	39055.41	9.5608546	9.5916812
	30	36650.13	39391.05	9 5640754	9.5953975
	40	36920.62	39727.46	9.5672689	9.5990908
	50	37190 80	40064.65	9.5704355	9.6027613
22	0	37460.66	40402.62	9.5735754	9.6064066
	10	37730.21	40741.39	9.5766892	9.6100359
	20	37999 44	41080.97	9 5797772	9.6136407
	30	38268.34	41421.38	9.5828397	9.6172243
	40	38536.93	41762 57	9.5858771	9.6207872
	50	38805.18	42104 60	9.5888897	9 6243296
23	0	39073.11	42447.49	9.5918780	9.6278519
	10	39340.71	42791.20	9.5948422	9.6313545
	20	39607.98	43135.79	9 5977827	9.6348378
	30	39874.91	43481.24	9.6006997	9 6383019
	40	40141 50	43827.56	9.6035936	9 6417473
	50	40407 75	44174.76	9.6064647	9 6451743

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
71	60	95105.65	307768.35	9.97782063	10.4882240
	50	95015.36	304749.15	9.9777938	10.4839425
	40	94924.26	301783.01	9.9773772	10.4796948
	30	94832.36	298868.50	9.9769566	10.4754801
	20	94739.66	296004.22	9.9765318	10.4712979
	10	94646.16	293188.85	9.9761030	10.4671474
70	60	94551.85	290421.09	9.9756701	10.4630281
	50	94456.75	287699.70	9.9752330	10.4589394
	40	94360.85	285023.49	9.9747918	10.4548807
	30	94264.15	282391.29	9.9743466	10.4508513
	20	94166.65	279801.98	9.9738971	10.4468508
	10	94068.35	277254.48	9.9734435	10.4428786
69	60	93969.26	274747.74	9.9729858	10.4389341
	50	93869.37	272280.75	9.9725239	10.4350169
	40	93768.69	269852.54	9.9720579	10.4311265
	30	93667.22	267462.15	9.9715876	10.4272623
	20	93564.95	265108.67	9.9711132	10.4234239
	10	93461.89	262791.21	9.9706346	10.4196108
68	60	93358.04	260508.91	9.9701517	10.4158226
	50	93253.40	258260.94	9.9696647	10.4120587
	40	93147.97	256046.49	9.9691734	10.4083188
	30	93041.75	253864.79	9.9686779	10.4046025
	20	92934.75	251715.07	9.9681781	10.4009092
	10	92826.96	249596.61	9.9676741	10.3972387
67	60	92718.39	247508.69	9.9671659	10.3935904
	50	92609.03	245450.61	9.9666533	10.3899641
	40	92498.88	243421.72	9.9661365	10.3863593
	30	92387.95	241421.36	9.9656153	10.3827757
	20	92276.24	239448.89	9.9650899	10.3792128
	10	92163.75	237503.72	9.9645602	10.3756704
66	60	92050.49	235585.24	9.9640261	10.3721481
	50	91936.44	233692.87	9.9634877	10.3686455
	40	91821.61	231826.06	9.9629449	10.3651622
	30	91706.01	229984.25	9.9633978	10.3616981
	20	91589.63	228166.93	9.9618463	10.3582527
	10	91472.47	226373.57	9.9612904	10.3548257

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
24	0	40673.66	44522.87	9.6093133	9.6485831
	10	40939.23	44871.87	9.6121397	9.6519742
	20	41204.46	45221.79	9.6149441	9.6553477
	30	41469.32	45572.64	9.6177270	9.6587041
	40	41733.85	45924.39	9.6204884	9.6620434
	50	41998.01	46277.09	9.6232287	9.6653662
25	0	42261.83	46630.77	9.6259483	9.6686725
	10	42525.28	46985.39	9.6286478	9.6719628
	20	42788.38	47340.98	9.6313258	9.6752372
	30	43051.11	47697.55	9.6339844	9.6784961
	40	43313.48	48055.12	9.6366231	9.6817396
	50	43575.48	48413.68	9.6392422	9.6849681
26	0	43837.12	48773.26	9.6418420	9.6881818
	10	44098.38	49133.86	9.6444226	9.6913809
	20	44359.27	49495.49	9.6469844	9.6945656
	30	44619.78	49858.16	9.6495274	9.6977363
	40	44879.92	50221.89	9.6520521	9.7008930
	50	45139.68	50586.68	9.6545584	9.7040362
27	0	45399.05	50952.54	9.6570468	9.7071659
	10	45658.04	51319.50	9.6595173	9.7102824
	20	45916.64	51687.55	9.6619701	9.7133859
	30	46174.86	52056.70	9.6644056	9.7164767
	40	46432.69	52426.98	9.6668238	9.7195549
	50	46690.12	52798.39	9.6692250	9.7226207
28	0	46947.16	53170.94	9.6716093	9.7256744
	10	47203.80	53544.65	9.6739769	9.7287161
	20	47460.04	53919.52	9.6763281	9.7317460
	30	47715.88	54295.57	9.6786629	9.7347644
	40	47971.31	54672.81	9.6809816	9.7377714
	50	48226.34	55051.25	9.6832843	9.7407672
29	0	48480.96	55430.90	9.6855712	9.7437520
	10	48735.17	55811.79	9.6878425	9.7467259
	20	48988.97	56193.91	6.6900983	9.7496892
	30	49242 36	56577.28	9.6923388	9.7526420
	40	49495.33	56961.91	9.6945642	9.7555846
	50	49747.87	57347.83	9.6967745	9.7585170

G.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
1					
65	60	91354.54	224603.68	9.9607302	10.3514169
	50	91235.84	222856.75	9.9601655	10.3480258
	40	91116.37	221132.34	9.9595964	10.3446523
	30	90996.13	219429.97	9.9590229	10.3412960
	20	90875.11	217749.20	9.9584450	10.3379566
	10	90753.33	216089.58	9.9578626	10.3346338
1					
64	60	90630.78	214450.69	9.9572757	10.3313275
	50	90507.46	212832.13	9.9566844	10.3280372
	40	90383.38	211233.48	9.9560886	10.3247628
	30	90258.53	209654.36	9.9554882	10.3215039
	20	90132.91	208094.38	9.9548834	10.3182604
	10	90006.54	206553.18	9.9542741	10.3150319
1					
63	60	89879.40	205030.38	9.9536602	10.3118182
	50	89751.51	203525.65	9.9530418	10.3086191
	40	89622.85	202038.62	9.9524188	10.3054344
	30	89493.43	200568.97	9.9517912	10.3022637
	20	89363.27	199116.37	9.9511590	10.2991070
	10	89232.33	197680.50	9.9505223	10.2959638
1					
62	60	89100.65	196261.05	9.9498809	10.2928341
	50	88968.21	194857.71	9.9492349	10.2897176
	40	88835.02	193470.20	9.9485842	10.2866141
	30	88701.08	192098.21	9.9479289	10.2835233
	20	88566.39	190741.47	9.9472689	10.2804451
	10	88430.95	189399.71	9.9466043	10.2773793
1					
61	60	88294.76	188072.65	9.9459349	10.2743256
	50	88157.82	186760.03	9.9452609	10.2712839
	40	88020.14	185461.59	9.9445821	10.2682540
	30	87881.71	184177.09	9.9438985	10.2652356
	20	87742.54	182906.28	9.9432102	10.2622286
	10	87602.62	181648.92	9.9425171	10.2592328
1					
60	60	87461.97	180404.78	9.9418193	10.2562480
	50	87320.58	179173.61	9.9411166	10.2532741
	40	87178.44	177955.24	9.9404091	10.2503108
	30	87035.57	176749.40	9.9396968	10.2473580
	20	86891.96	175555.90	9.9389796	10.2444154
	10	86747.62	174374.53	9.9382576	10.2414830

Gr.	Min.	Sinus.	Tangens.	Leg. Sin.	Log. Tang.
30	0	50000.00	57735.03	9.6989700	9.7614394
	10	50251.70	58123.53	9.7011508	9.7643520
	20	50502.99	58513.35	9.7033170	9.7672550
	30	50753.84	58904.50	9.7054689	9.7701485
	40	51004.26	59296.99	9.7076064	9.7730327
	50	51254.25	59690.84	9.7097299	9.7759077
31	0	51503.81	60086.86	9.7118393	9.7787737
	10	51752.93	60482.66	9.7139349	9.7816309
	20	52001.61	60880.67	9.7160168	9.7844794
	30	52249.86	61280.08	9.7180851	9.7873193
	40	52497.66	61680.92	9.7201399	9.7901508
	50	52745.02	62083.20	9.7221814	9.7929741
32	0	52991.93	62486.94	9.7242097	9.7957892
	10	53238.39	62892.15	9.7262249	9.7985964
	20	53484.40	63298.83	9.7282271	9.8013957
	30	53729.96	63707.03	9.7302165	9.8041873
	40	53975.07	64116.78	9.7321932	9.8069714
	50	54219.71	64527.97	9.7341572	9.8097480
33	0	54463.90	64940.76	9.7361088	9.8125174
	10	54707.63	65355.11	9.7380479	9.8152795
	20	54950.90	65771.03	9.7399748	9.8180347
	30	55193.70	66188.56	9.7418895	9.8207829
	40	55436.03	66607.69	9.7437921	9.8235244
	50	55677.90	67028.45	9.7456828	9.8262592
34	0	55919.29	67450.85	9.7475617	9.8289874
	10	56160.21	67874.92	9.7494287	9.8317093
	20	56400.65	68300.66	9.7512842	9.8344249
	30	56640.62	68728.10	9.7531280	9.8371343
	40	56880.11	69157 24	9.7549604	9.8398377
	50	57119.12	69588.13	9.7567815	9.8425351
35	0	57357.64	70020.75	9.7585913	9.8452268
	10	57595.68	70455.15	9.7603899	9.8479127
	20	57833.23	70891.33	9.7621775	9.8505931
	30	58070.30	71329.31	9.7639540	9.8532680
	40	58306.87	71769.11	9.7657197	9.8559376
	50	58542.94	72210.75	9.7674746	9.8586019

°	′	Sinus.	Tangens.	Log. sin.	Log. Tang.
59	60	86602.54	173205.08	9.93775306	10.23856066
	50	86456.73	172047.36	9.93679888	10.23564806
	40	86310.19	170901.16	9.93602618	10.23274506
	30	86162.92	169766.31	9.93532048	10.22985156
	20	86014.91	168642.61	9.93457388	10.22696736
	10	85866.18	167529.88	9.93382228	10.22409236
58	60	85716.73	166427.95	9.93306568	10.22122636
	50	85566.55	165336.63	9.93230408	10.21836916
	40	85415.64	164255.76	9.93153748	10.21552066
	30	85264.02	163185.17	9.93076588	10.21268076
	20	85111.66	162124.69	9.92998918	10.20984926
	10	84958.60	161074.17	9.92920738	10.20702596
57	60	84804.81	160033.45	9.92842058	10.20421086
	50	84650.30	159002.38	9.92762858	10.20140366
	40	84495.08	157980.79	9.92683148	10.19860436
	30	84339.12	156968.56	9.92602928	10.19581276
	20	84182.49	155965.52	9.92522188	10.19302866
	10	84025.13	154971.55	9.92440928	10.19025206
56	60	83867.06	153986.59	9.92359148	10.18748276
	50	83708.27	153010.23	9.92276848	10.18472056
	40	83548.78	152042.61	9.92194018	10.18196536
	30	83388.58	151083.52	9.92110668	10.17921716
	20	83227.68	150132.82	9.92026778	10.17647566
	10	83066.07	149190.38	9.91942378	10.17374086
55	60	82903.76	148256.10	9.91857428	10.17101256
	50	82740.74	147329.83	9.91771948	10.16829076
	40	82577.03	146411.47	9.91685938	10.16557516
	30	82412.62	145500.90	9.91599378	10.16286576
	20	82247.51	144598.01	9.91512288	10.16016236
	10	82081.70	143702.68	9.91424648	10.15746496
54	60	81915.21	142814.80	9.91336458	10.15477326
	50	81748.01	141934.27	9.91247728	10.15208736
	40	81580.13	141060.98	9.91158448	10.14940696
	30	81411.55	140194.83	9.91068608	10.14673206
	20	81242.29	139335.71	9.90978218	10.14406246
	10	81072.33	138483.53	9.90887278	10.14139816

Gr.	Min.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
36	0	58778.53	72654.26	9.7692187	9.8612610
	10	59013.61	73099.63	9.7709522	9.8639152
	20	59248.19	73546.91	9.7726751	9.8665644
	30	59482.28	73996.11	9.7743876	9.8692089
	40	59715.86	74447.24	9.7760897	9.8718486
	50	59948.93	74900.33	9.7777815	9.8744838
37	0	60181 50	75355.41	9.7794630	9.8771144
	10	60413.56	75812.48	9.7811344	9.8797407
	20	60645.11	76271.57	9.7827958	9.8823627
	30	60876.14	76732.70	9.7844471	9.8849805
	40	61106.66	77195.89	9.7860886	9.8875942
	50	61336.66	77661.17	9.7877202	9.8902040
38	0	61566.15	78128.56	9.7893420	9.8928098
	10	61795.11	78598.08	9.7909541	9.8954119
	20	62023.55	79069.75	9.7925566	9.8980104
	30	62251.46	79543.59	9.7941496	9.9006052
	40	62478.85	80019.63	9.7957330	9.9031966
	50	62705.71	80497.90	9.7973071	9.9057845
39	0	62932 04	80978.40	9.7988718	9.9083692
	10	63157.84	81461.18	9.8004272	9.9109507
	20	63383.09	81946.25	9.8019735	9.9135291
	30	63607.82	82433.64	9.8035105	9.9161045
	40	63842.01	82923.37	9.8050385	9.9186769
	50	64055.66	83415.47	9.8065575	9.9212466
40	0	64278.76	83909.96	9.8080675	9.9238135
	10	64501.32	84406.88	9.8095686	9.9263778
	20	64723.34	84906.24	9.8110609	9.9289396
	30	64944.80	85408.07	9.8125444	9.9314989
	40	65165.72	85912.40	9.8140192	9.9340559
	50	65386.09	86419.26	9.8154854	9.9366105
41	0	65605.90	86928.68	9.8169429	9.9391631
	10	65825.16	87440.67	9.8183919	9.9417135
	20	66043.86	87955.28	9.8198325	9.9442619
	30	66262.01	88472.53	9.8212646	9.9468084
	40	66479.59	88992.45	9.8226883	9.9493531
	50	66696.61	89515.06	9.8241037	9.9518961

°	′	Sinus.	Tangens.	Log. Sin.	Log. Tang.
53	60	80901.70	137638.19	9.9079576	10.1387390
	50	80730.38	136799.59	9.9070370	10.1360848
	40	80558.37	135967.64	9.9061107	10.1334356
	30	80385.69	135142.24	9.9051787	10.1307911
	20	80212.32	134323.31	9.9042411	10.1281514
	10	80038.27	133510.75	9.9032977	10.1255162
52	60	79863.55	132704.48	9.9023486	10.1228856
	50	79688.15	131904.41	9.9013938	10.1202593
	40	79512.08	131110.46	9.9004331	10.1176373
	30	79335.33	130322.54	9.8994667	10.1150195
	20	79157.92	129540.57	9.8984944	10.1124058
	10	78979.83	128764.47	9.8975162	10.1097960
51	60	78801.07	127994.16	9.8965321	10.1071902
	50	78621.65	127229.57	9.8955422	10.1045881
	40	78441.57	126470.62	9.8945463	10.1019896
	30	78260.82	125717.23	9.8935444	10.0993948
	20	78079.40	124969.33	9.8925364	10.0968034
	10	77897.33	124226.85	9.8915226	10.0942155
50	60	77714.60	123489.72	9.8905026	10.0916308
	50	77531.21	122757.86	9.8894765	10.0890493
	40	77347.16	122031.21	9.8884444	10.0864709
	30	77162.46	121309.70	9.8874061	10.0838955
	20	76977.10	120593.27	9.8863616	10.0813231
	10	76791.10	119881.84	9.8853109	10.0787534
49	60	76604.44	119175.36	9.8842540	10.0761865
	50	76417.14	118473.76	9.8831908	10.0736222
	40	76229.19	117776.98	9.8821213	10.0710604
	30	76040.60	117084.96	9.8810455	10.0685011
	20	75851.36	116397.63	9.8799634	10.0659441
	10	75661.47	115714.95	9.8788748	10.0633895
48	60	75470.97	115036.84	9.8777799	10.0608369
	50	75279.80	114363.26	9.8766785	10.0582865
	40	75088.00	113694.14	9.8755706	10.0557381
	30	74895.57	113029.44	9.8744561	10.0531916
	20	74702.51	112369.09	9.8733352	10.0506469
	10	74508.81	111713.05	9.8722076	10.0481039

G.	M.	Sinus.	Tangens.	Log. Sin.	Log. Tang.
42	0	66913.06	90040.41	9.8255109	9.9544378
	10	67128.95	90568.51	9.8269098	9.9569772
	20	67344.27	91099.41	9.8283006	9.9595155
	30	67559.02	91633.12	9.8296833	9.9620525
	40	67773.20	92169.68	9.8310589	9.9645881
	50	67986.81	92709.14	9.8324246	9.9671225
43	0	68199.84	93251.51	9.8337833	9.9696559
	10	68412.29	93796.83	9.8351341	9.9721882
	20	68624.16	94345.13	9.8364771	9.9747195
	30	68835.45	94896.46	9.8378122	9.9772500
	40	69046.17	95450.83	9.8391396	9.9797797
	50	69256.30	96008.29	9.8404593	9.9823087
44	0	69465.84	96568.88	9.8417713	9.9848372
	10	69674.79	97132.62	9.8430757	9.9873651
	20	69883.15	97699.56	9.8443725	9.9898926
	30	70090.93	98269.73	9.8456618	9.9924197
	40	70298.10	98843.16	9.8469436	9.9949466
	50	70504.69	99419.91	9.8482180	9.9974734
45	0	70710.68	100000.00	9.8494850	10.0000000

In Tabulis his Secantes prætermisimus, tum quia in ea, quam secuti sumus, methodo nunquam adhibentur: tum quia notis complementorum sinubus nullo negotio possunt investigari.

Dato Sinu complementi cujuslibet arcus vel anguli, invenire illius Secantem.

Divide quadratum Sinûs totius per Sinum complementi; quotiens erit Secans quæsita. *Ratio patet ex p. 2. c. 1. p. 1.*

Exemplum. Quæratur Secans anguli 60 gr. Quadratum radii, seu 100000000000000, divide per 5000000 sinum 30 gr. seu complementi. Quotiens 20000000 erit Secans quæsita.

Si per Logarithmos opereris, ex Logarithmo radii duplicato, subtrahe Logarithmum Sinûs complementi. Residuum erit Secans quæsita.

Idem obiter hic dicendo: Ut sinus anguli dati, ad sinum totum: sic tangens anguli dati, ad secantem quæsitam.

°	′	Sinus.	Tangens.	Log. Sin.	Log. Tang.
47	60	7431448	11110611.25	9.8710735	10.0455626
	50	7411959.53	11041315.65	9.8699326	10.0430228
	40	7392394	10977010.20	9.8687851	10.0404849
	30	7372773	10913086.85	9.8676309	10.0379478
	20	7353090	10849554	9.8664699	10.0354119
	10	7333345	10786423	9.8653021	10.0328775
46	60	7313537	10723687	9.8641275	10.0303441
	50	7293667	10661341	9.8629460	10.0278118
	40	7273736	10599381	9.8617576	10.0252805
	30	7253744	10537808	9.8605622	10.0227500
	20	7233690	10476595	9.8593599	10.0202203
	10	7213574	10415767	9.8581505	10.0176913
45	60	7193398	10355303	9.8569341	10.0151628
	50	7173161	10295203	9.8557106	10.0126349
	40	7152863	10235461	9.8544799	10.0101074
	30	7132505	10176074	9.8532421	10.0075803
	20	7112086	10117037	9.8519970	10.0050534
	10	7091607	10058347	9.8507446	10.0025268
44	60	7071068	10000000	9.8494850	10.0000000

Moneri hîc etiam debet Lector, quòd tametsi in Tabulis computari soleant Sinus, Tangentes & Secantes pro Sinu toto 10000000: abjectis tamen duabus notis prioribus, habeantur Sinus &c. ad radium 100000, quales ad operationes obviàs satis accuratè peragendas omninò sufficiunt. Quænam verò notæ sint abjiciendæ, indicat punctum quod postponi solet duabus primis notis à dextra vel expressis vel intellectis.

Dixi *expressis vel intellectis*; quia si punctum, ut quandoque fit, sequatur immediatè primam notam, indicium est deesse unam: si verò præfigatur primæ, ostendit desiderari duas notas, quibus totidem Cyphræ sunt substituendæ. *Ex. gr.* Si in Tabulis deprehendatur Tangens 88143 57. 1, adjici debet una Cyphra, sic 88143 57. 10: vel si inveniatur Secans 85943689. præfigendæ sunt duæ Cyphræ, sic 85943689.00, ut habeatur Tangens vel Secans integra pro radio 10000000.

TABULA LOGARITHMORUM,

Pro Numeris naturali ſerie creſcentibus ab Unitate ad 1000.

QUæ ad Logarithmorum naturam, inventionem, uſumque pertinent, videre poterit ſtudioſus Lector in Arithmetica methodicè ſuccinctèque tradita Cap. 7. ubi (§. 3. & 4.) demonſtratur modus facilis inveniendi cujuſcunquè numeri, ſive fracti, ſive integri Logarithmum; & (§. 5.) numerum cuicunque Logarithmo dato correſpondentem. Nec refert quòd Regulæ ibidem traditæ adaptentur præcipuè Tabulis majoribus continentibus Logarithmos numerorum naturali ſerie creſcentium ab unitate ad 10000, quales paſſim extant; nam quicunque Regulas illas, & illarum demonſtrationes intellexerit, quæ iſtic præcipiuntur, ſequenti Tabulæ facilè applicabit.

Aliud etiam eſt, quod ſi à Lectore obſervetur, Tabulæ hujus Logarithmicæ brevitatem aliquatenùs compenſabit; ſi nimirum quando numerus proponitur excedens maximum in Tabula, eum (*per Reductionis Regulam*) convertat in alium minorem, ſed æquivalentem in ſpecie diverſa: *Ex. gr.* Si in triangulo aliquo reſolvendo detur latus continens pedes 2400, reducantur pedes ad ulnas 800, vel paſſus 480, vel hexapedas 400, vel decempedas 240. Et per numeri ſic reducti Logarithmum perficiatur operatio.

Quòd ſi contingat ut poſt reductionem ſuperſit aliqua fractio, quomodò inveniatur Logarithmus huic correſpondens in Arithmetica indicatur.

N.	Logarith.
1	0.0000000
2	0.3010300
3	0.4771212
4	0.6020600
5	0.6989700
6	0.7781512
7	0.8450980
8	0.9030900
9	0.9542425
10	1.0000000
11	1.0413927
12	1.0791812
13	1.1139433
14	1.1461280
15	1.1760913
16	1.2041200
17	1.2304489
18	1.2552725
19	1.2787531
20	1.3010300
21	1.3222193
22	1.3424227
23	1.3617278
24	1.3802112
25	1.3979400
26	1.4149733
27	1.4313638
28	1.4471580
29	1.4623980
30	1.4771212
31	1.4913617
32	1.5051500
33	1.5185139
34	1.5314789
35	1.5440680
36	1.5563025
37	1.5682017
38	1.5797836
39	1.5910646
40	1.6020600
41	1.6127839
42	1.6232493
43	1.6334685
44	1.6434527
45	1.6532125
46	1.6627578
47	1.6720979
48	1.6812412
49	1.6901961
50	1.6989700
51	1.7075702
52	1.7160033
53	1.7242759
54	1.7323938
55	1.7403627
56	1.7481880
57	1.7558748
58	1.7634280
59	1.7708520
60	1.7781512
61	1.7853298
62	1.7923917
63	1.7993405
64	1.8061800
65	1.8129133
66	1.8195439
67	1.8260748
68	1.8325089
69	1.8388491
70	1.8450980
71	1.8512583
72	1.8573325
73	1.8633229
74	1.8692317
75	1.8750613
76	1.8808136
77	1.8864907
78	1.8920946
79	1.8976271
80	1.9030900
81	1.9084850
82	1.9138138
83	1.9150781
84	1.9242793
85	1.9294189
86	1.9344984
87	1.9395192
88	1.9444827
89	1.9493900
90	1.9542425
91	1.9590414
92	1.9637878
93	1.9684829
94	1.9731278
95	1.9777236
96	1.9822712
97	1.9867717
98	1.9912761
99	1.9956352

100

N.	Logarith.	N.	Logarith.	N.	Logarith.
101	2.0043214	134	2.1271048	167	2.2227165
102	2.0086602	135	2.1303338	168	2.2253093
103	2.0128372	136	2.1335389	169	2.2278867
104	2.0170333	137	2.1367206	170	2.2304489
105	2.0211893	138	2.1398791	171	2.2329961
106	2.0253059	139	2.1430148	172	2.2355284
107	2.0293838	140	2.1461280	173	2.2380461
108	2.0334238	141	2.1492191	174	2.2405492
109	2.0374265	142	2.1522883	175	2.2430380
110	2.0413927	143	2.1553360	176	2.2455127
111	2.0453230	144	2.1583625	177	2.2479733
112	2.0492180	145	2.1613680	178	2.2504200
113	2.0530784	146	2.1643528	179	2.2528530
114	2.0569048	147	2.1673173	180	2.2552725
115	2.0606978	148	2.1702617	181	2.2576786
116	2.0644580	149	2.1731863	182	2.2600714
117	2.0681859	150	2.1760913	183	2.2624511
118	2.0718820	151	2.1789769	184	2.2648178
119	2.0755470	152	2.1818436	185	2.2671717
120	2.0791812	153	2.1846914	186	2.2695129
121	2.0827854	154	2.1875207	187	2.2718416
122	2.0863598	155	2.1903317	188	2.2741578
123	2.0899051	156	2.1931246	189	2.2764618
124	2.0934217	157	2.1958996	190	2.2787536
125	2.0969100	158	2.1986571	191	2.2810334
126	2.1003704	159	2.2013971	192	2.2833012
127	2.1038037	160	2.2041200	193	2.2855573
128	2.1072100	161	2.2068259	194	2.2878017
129	2.1105897	162	2.2095150	195	2.2900346
130	2.1139433	163	2.2121876	196	2.2922561
131	2.1172713	164	2.2148438	197	2.2944662
132	2.1205739	165	2.2174839	198	2.2966652
133	2.1238516	166	2.2201081	199	2.2988531
134	2.1271048	167	2.2227165	200	2.3010300

N.	Logarith.	N.	Logarith.	N.	Logarith.
200					
201	2.3031961	234	2.3692159	267	2.4265113
202	2.3053514	235	2.3710679	268	2.4281348
203	2.3074960	236	2.3729120	269	2.4297523
204	2.3096302	237	2.3747483	270	2.4313638
205	2.3117539	238	2.3765770	271	2.4329693
206	2.3138672	239	2.3783979	272	2.4345689
207	2.3159703	240	2.3802112	273	2.4361626
208	2.3180633	241	2.3820170	274	2.4377506
209	2.3201463	242	2.3838154	275	2.4393327
210	2.3222193	243	2.3856063	276	2.4409091
211	2.3242824	244	2.2873898	277	2.4424798
212	2.3263359	245	2.3891661	278	2.4440448
213	2.3283796	246	2.3909351	279	2.4456042
214	2.3304138	247	2.3926969	280	2.4471580
215	2.3324385	248	2.3944517	281	2.4487063
216	2.3344537	249	2.3961993	282	2.4502491
217	2.3364597	250	2.3979400	283	2.4517864
218	2.3384565	251	2.3996737	284	2.4533183
219	2.3404441	252	2.4014005	285	2.4548449
220	2.3424227	253	2.4031205	286	2.4563660
221	2.3443923	254	2.4048337	287	2.4578819
222	2.3463530	255	2.4065402	288	2.4593925
223	2.3483049	256	2.4082400	289	2.4608978
224	2 3502480	257	2.4099331	290	2.4623980
225	2.3521825	258	2.4116197	291	2.4638930
226	2.3541084	259	2.4132998	292	2.4653828
227	2.3560259	260	2.4149733	293	2.4668676
228	2.3579348	261	2.4166405	294	2.4683473
229	2.3598355	262	2.4183013	295	2.4698220
230	2.3617278	263	2.4199557	296	2.4712917
231	2.3636120	264	2.4216039	297	2.4727564
232	2.3654880	265	2.4232459	298	2.4742163
233	2.3673559	266	2.4248816	299	2.4756712
234	2.3692159	267	2.4265113	300	2.4771212

N.	Logarith.	N.	Logarith.	N.	Logarith.
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302	2.4800069	335	2.5250448	368	2.5658478
303	2.4814426	336	2.5263393	369	2.5670264
304	2.4828736	337	2.5276299	370	2.5682017
405	2.4842998	338	2.5289167	371	2.5693739
306	2.4857214	339	2.5301997	372	2.5705429
307	2.4871384	340	2.5314789	373	2.5717088
308	2.4885507	341	2.5327544	374	2.5728716
309	2.4899585	342	2.5340261	375	2.5740313
310	2.4913617	343	2.5352941	376	2.5751878
311	2.4927604	344	2.5365584	377	2.5763413
312	2.4941546	345	2.5378191	378	2.5774918
313	2.4955443	346	2.5390761	379	2.5786392
314	2.4969296	347	2.5403295	380	2.5797836
315	2.4983105	348	2.5415792	381	2.5809250
316	2.4996871	349	2.5428254	382	2.5820634
317	2.5010593	350	2.5440680	383	2.5831988
318	2.5024271	351	2.5453071	384	2.5843312
319	2.5037907	352	2.5465427	385	2.5854607
320	2.5051500	353	2.5477747	386	2.5865873
321	2.5065050	354	2.5490033	387	2.5877110
322	2.5078559	355	2.5502283	388	2.5888317
323	2.5092025	356	2.5514500	389	2.5899496
324	2.5105450	357	2.5526682	390	2.5910646
325	2.5118834	358	2.5538830	391	2.5921768
326	2.5132176	359	2.5550944	392	2.5932861
327	2.5145477	360	2.5563025	393	2.5943925
328	2.5158738	361	2.5575072	394	2.5954962
329	2.5171959	362	2.5587086	395	2.5965971
330	2.5185139	363	2.5599066	396	2.5976952
331	2.5198280	364	2.5611014	397	2.5987905
332	2.5211381	365	2.5622929	398	2.5998831
333	2.5224442	366	2.5634811	399	2.6009729
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413	2.6159500
414	2.6170003
415	2.6180481
416	2.6190933
417	2.6201360
418	2.6211763
419	2.6222140
420	2.6232493
421	2.6242821
422	2.6253124
423	2.6263404
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455	2.6580114
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457	2.6599162
458	2.6608655
459	2.6618127
460	2.6627578
461	2.6637009
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472	2.6739420
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474	2.6757783
475	2.6766936
476	2.6776069
477	2.6785184
478	2.6794279
479	2.6803355
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482	2.6830470
483	2.6839471
484	2.6848454
485	2.6857417
486	2.6866363
487	2.6875290
488	2.6884198
489	2.6893089
490	2.6901961
491	2.6910815
492	2.6919651
493	2.6928469
494	2.6937269
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497	2.6963564
498	2.6972293
499	2.6981005
500	2.6989700

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543	2.7347998
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547	2.7379873
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557	2.7458552
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582	2.7649230
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604	2.7810369	637	2.8041394	670	2.8260748
605	2.7817554	638	2.8048207	671	2.8267225
606	2.7824726	639	2.8055009	672	2.8273693
607	2.7831887	640	2.8061800	673	2.8280151
608	2.7839036	641	2.8068580	674	2.8286599
609	2.7846173	642	2.8075350	675	2.8293038
610	2.7853298	643	2.8082110	676	2.8299467
611	2.7860412	644	2.8088859	677	2.8305887
612	2.7867514	645	2.8095597	678	2.8312297
613	2.7874605	646	2.8102325	679	2.8318698
614	2.7881684	647	2.8109043	680	2.8325089
615	2.7888751	648	2.8115750	681	2.8331471
616	2.7895807	649	2.8122447	682	2.8337844
617	2.7902852	650	2.8129134	683	2.8344207
618	2.7909885	651	2.8135810	684	2.8350561
619	2.7916906	652	2.8142476	685	2.8356906
620	2.7923917	653	2.8149132	686	2.8363241
621	2.7930916	654	2.8155777	687	2.8369567
622	2.7937904	655	2.8162413	688	2.8375884
623	2.7944880	656	2.8169038	689	2.8382192
624	2.7951846	657	2.8175654	690	2.8388491
625	2.7958800	658	2.8182259	691	2.8394780
626	2.7965744	659	2.8188854	692	2.8401061
627	2.7972675	660	2.8195439	693	2.8407332
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629	2.7986506	662	2.8208580	695	2.8419848
630	2.7993405	663	2.8215135	696	2.8426092
631	2.8000294	664	2.8221681	697	2.8432328
632	2.8007171	665	2.8228216	698	2.8438554
633	2.8014037	666	2.8234742	699	2.8444772
634	2.8020893	667	2.8241258	700	2.8450980

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709	2.8506462
710	2,8512583
711	2.8518696
712	2.8524800
713	2.8530895
714	2,8536982
715	2.8543060
716	2.8549130
717	2,8555191
718	2.8561244
719	2.8567289
720	2.8573325
721	2.8579353
722	2.8585372
723	2.8591383
724	2.8597386
725	2.8603380
726	2.8609366
727	2.8615344
728	2.8621314
729	2 8627275
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731	2.8639174
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740	2.8692317
741	2.8698182
742	2.8704039
743	2.8709888
744	2.8715729
745	2.8721563
746	2.8727388
747	2.8733206
748	2.8739016
749	2.8744818
750	2.8750613
751	2,8756399
752	2,8762178
753	2.8767950
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780	2.8920946
781	2.8926510
782	2.8932067
783	2.8937618
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787	2.8959747
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855	2.9319661
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857	2.9329808
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875	2.9420080
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PRÆCIPUA GEOMETRIÆ PLANÆ ELEMENTA

EX EUCLIDE SELECTA

ET

Incipientium uſui accommodata.

Tametsi in Propositionum delectu præcipua cura fuit, ne qua deesset Trigonometriæ demonstrandæ: simul tamen dedi operam, ut Opus hoc ad alia quàm plurima inserviret; ideoque adjunxi ea, quæ videbantur ad reliquam Matheſim maximè usui futura. Quoad Propositionum ordinem, eum tenui, qui ab Euclide ejusque Interpretibus observatur. Corollaria verò adjeci illa quæ videbantur incipientibus magis necessaria.

EX LIBRO PRIMO.

DEFINITIONES.

1. *Punctum* est magnitudinis initium nullam habens dimensionem.

2. *Linea* est magnitudo solam habens longitudinem.

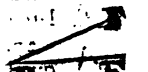
4. *Linea Recta* est brevissima quæ inter duo puncta poni potest duci.

5. *Superficies* est magnitudo tantùm longa & lata.

7. *Superficies Plana* est minima omnium eosdem terminos habentium.

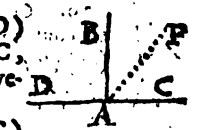
Corpus est magnitudo longa, lata, & profunda.

8. *Angulus* est duarum linearum (BE, BF) se mutuò tangentium, alterius ad alterum inclinatio, seu (ut alii malunt) alterius ab altero divaricatio: dicitur *Rectilineus*, *Curvilineus*, vel *Mixtilineus*, pro diversitate linearum quibus efficitur. Hæ lineæ (BE, BF) angulum continentes vocantur ejus *Crura*; punctum verò (B) in quo crura sibi mutuò occurrunt, est *Vertex* anguli (EBF.)



Quando angulus designatur tribus literis (EBF) media (B) denotat ipsum angulum.

14. Recta (AB) ita insistens alteri (CD) ut faciat utrinque æquales angulos (BAC, BAD) dicitur *Perpendicularis*; angulus verò à tali linea factus, est *Rectus*.



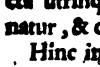
15. Angulus (FAC) minor recto (BAC) est *Acutus*; Angulus (FAD) major recto (BAD) est *obtusus*: uterque verò *Obliquus*.

18. *Circulus* est superficies plana comprehensa circuitu unius lineæ, quæ *Circumferentia* dicitur, ad quam omnes rectæ ductæ ab aliquo puncto (A quod *Centrum* vocatur) intra circulum posito, sunt æquales.

20. *Diameter* circuli est recta (BD) quæ per centrum ducta utrinque ad circumferentiam, seu peripheriam, terminatur, & circulum bifariam dividit.

Hinc intelligitur quid sit *Semicirculus*, *Semidiameter*, seu *Radius* circuli, & *Semicircumferentia*.

23. *Rectilinea Figura* est superficies plana rectis lineis undique terminata.



24. *Triangulum* sive *Trilaterum*, est superficies plana (ABC) tribus lineis rectis (AB, BC, CA) comprehensa.

FIG. TRILATERÆ.

25. *Triang. Æquilaterum* (A) est quod habet tria latera æqualia.

26. *Isosceles* (B) est quod habet duo saltem latera æqualia.

27. *Scalenum* (C) est quod habet tria latera inæqualia.

28. *Triang. Rectangulum* est cujus unus angulus (D) est rectus.

29. *Obtusangulum* est cujus unus angulus (E) est obtusus.

30. *Acutangulum* (F) quod habet angulos tres acutos.

FIG. QUADRILATERÆ.

31. *Rectangulum* est Fig. quadril. (G vel H) habens 4 angulos rect.

32. *Quadratum* (H) est Rectangulum æquilaterum.

33. *Rhombus* (I) est, qui est æquilaterus, non æquiangulus.

34. *Rhomboides* (K) est, qui opposita tantùm latera & angulos habet æqualia.

35. *Parallelogrämum.* (L) est cujus adversa latera sunt parallela.

Trapezium est quævis Fig. quadril. (M *v. g.*) præter prædictas.

36. Lineæ *Parallelæ* sunt, quæ in eodem plano existentes, quantumcumque utrinque protrahantur, semper distant æqualiter. Æqualis hæc distantia mensuratur per æqualia perpendicula (AB, CD)

38. Figuræ *Multilateræ* sive *Polygonæ*, sunt quæ pluribus quàm quatuor lateribus continentur.

AXIOMATA.

1. Quæ ſunt eidem tertio æqualia, ſunt æqualia inter ſe.
2. Si æqualibus addas vel demas æqualia, manebunt æqualia.
3. Quæ ſunt ejuſdem vel æqualium dimidia, dupla, ſubtripla, tripla, &c. ſunt inter ſe æqualia.
4. Quæ ſibi mutuò congruunt, ſunt æqualia.
5. Perpendicula (AB, CD) ex parallelis intercipiunt partes æquales (AC, BD.)

PROPOSITIO I.

Super datâ rectâ (AB) *conſtruere triangulum æquilaterum.*

Centris A & B, intervallo AB, deſcribe arcus ſe interſecantes in C, erit triangulum ABC æquilaterum; nam latera AC, BC (*ex conſtr.*) æquantur lateri AB.

PROPOSITIO IV.

Si triangula duo (X & Z) *habeant circa æquales angulos* (B & F) *latera* (BA, FL, *item* BC, FI) *ſibi mutuò æqualia; erunt reliqua omnia æqualia, viz. baſes* (AC, LI) *& reliqui ad baſim anguli* (A, L & C, I) *qui æqualibus lateribus oppoſuntur.*

Triangulo X concipiatur imponi triangulum Z. Quia anguli B & F ſunt æquales, latera LF, FI incident in latera BA, BC; ipſiſque, quia (*ex ſuppoſ.*) æqualia ſunt, perfectè congruent. Ergo puncta L & I

incident in A & C; adeoque baſis LI congruet baſi AC; ergo (*per ax.* 4) tota triangula X & Z ſunt æqualia.

Scholium.

Si baſes (AC, LI) *& anguli adjacentes* (A & C *ipſis* L & I) *fuerint æquales, tota triangula* (X & Z) *erunt æqualia.*

Nam ſi LI ponatur ſuper AC æquale, quia æquantur anguli L & A, I & C; latus FI incidet lateri BC, & latus FL lateri BA, & punctum F in B; adeoque omnia congruent, eruntque (*per ax.* 4) æqualia.

PROPOSITIO V.

In triangulo Isosceli anguli (A & C) ad basim, sunt æquales.

Concipe triangulum ABC iterum poni, sed seu converso, *cba*. In triangulis ABC, *cba*, latera AB, *cb*, item CB, *ab*, & anguli contenti B & *b* sunt (*ex suppos.*) æquales: ergo (*per 4. h*) anguli A & *c*, id est (quia c & C iidem sunt) A & C sunt æquales.

PROPOSITIO VI.

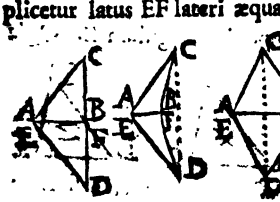
Si in triangulo anguli duo (A & C) sint æquales, latera (AB, CB) iis opposita sunt æqualia.

Convertatur triangulum ABC, ita ut fiat *cba*. In triangulis ABC, *cba*, latera AC, *ca*, & anguli adjacentes A & *c*, item C & *a* (*ex suppos.*) sunt æquales: ergo (*per schol. p. 4. h*) reliqua omnia sunt æqualia: ergo æquantur latera AB & *cb* seu CB.

PROPOSITIO VIII.

Si triangula (ABC, EFD) sint sibi mutuò æquilatera, etiam anguli æqualibus lateribus oppositi sunt æquales.

Æqualia sint latera AB & EF, AC & ED, BC & FD. Applicetur latus EF lateri æquali AB, sed ita ut circa eundem angulum DAC sint æqualia latera AC, ED; dein ducatur DC, & erunt (*per constr.*) triangula Isosceles DAC; & (nisi CD coincidat cum lateribus DF, BC ut in *Fig. 1.*) DBC: ergo si angulis ACD, ADC (*per 5. h*) æqualibus addas vel subtrahas (si opus sit) angulos BCD & BDC etiam (*per 5. h*) æquales; habebis angulos ACB, EDF æquales. Sed latera circa hos angulos viz. AC & ED, BC & FD (*ex suppos.*) sunt æqualia: ergo (*per 4. h*) reliqua sunt æqualia, & æquales sunt anguli æqualibus lateribus oppositi.



Dixi, *nisi CD coincidat cum lateribus* DF, BC, *ut in Fig. 1.* quia tunc habetur unum triangulum Isosceles DAC, cujus

anguli D & C (*per* 5. *h*) sunt æquales; unde in triangulis æquilateris ACB, EDF reliqui omnes (*per* 4. *h*) sunt æquales.

PROPOSITIO IX.

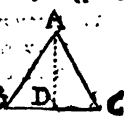
Bifariàm secare angulum datum (EAF.

Ex cruribus AE, AF accipe partes æquales AB, AC. Dein centris B & C, eodem radio, describe arcus sibi occurrentes in D: linea DA bisecabit angulum EAF. Ducantur enim BD, CD: Jam in triangulis BAD, CAD, latera AB, AC, & BD, CD sunt (*ex constr.*) æqualia, & latus AD commune: ergo (*per* 8. *h*) æquantur anguli BAD, CAD; adeoque bisectus est angulus EAF.

PROPOSITIO X.

Bisecare lineam datam (BC.)

Fiat (*per* 1. *h*) triangulum æquilaterum BAC, & (*per* 9 *h*) bisecetur angulus A per rectam AD, hæc bisecabit lineam BC. Nam in triangulis BAD, CAD, anguli BAD, CAD sunt (*ex constr.*) æquales, sicut & latera BA, CA; & latus AD commune: ergo (*per* 4. *h*) latera BD, CD sunt æqualia; adeoque bisecta est BC.



PROPOSITIO XI.

Ex puncto (O) *dato in linea recta excitare perpendiculum.*

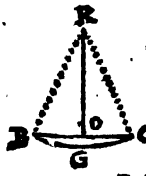
Accipe utrinque æquales OB, OC. Dein centris B & C, eodem intervallo describe arcus se secantes in A; erit AO perpendicularis ad BC.

Ducantur BA, CA.

Patet (*ex constr.*) triangula BAO, CAO esse sibi mutuò æquilatera: ergo (*per* 8. *h*) æquales sunt anguli AOB, AOC oppositi æqualibus lateribus BA, CA: ergo (*per* 14. *def. h*) AO perpendicularis est ad BC.

PROPOSITIO XII.

Ex puncto (R) *dato extra lineam* (BC) *demittere ad eam perpendiculum.*

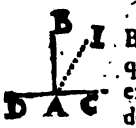


Centro R describatur arcus BGC secans rectam BC in B & C, & bisecetur BC in O; erit RO perpendicularis ad EF, Ducantur BR, CR.

Demonstratio est eadem quæ præcedentis.

PROPOSITIO XIII.

Recta insistens rectæ facit angulos æquales duobus rectis.

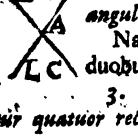


Nam si sit perpendicularis, ut BA, anguli BAD, BAC (*per 14. def. h*) sunt recti, Si obliquè insistat ut IA, angulus obtusus IAD tantùm excedit rectum BAD, quantùm acutus IAC deficit à recto BAC: ergo IAD, IAC simul æquantur duobus rectis BAD, BAC.

Corollaria.

1. *Quotcunque insistunt eidem rectæ, ad idem punctum & ad eandem partem faciunt angulos æquales duobus rectis.*

Nam sunt duo recti in plures obliquos divisi.



2. *Duæ rectæ* (BC, FE) *se intersecantes efficiunt angulos æquales quatuor rectis.*

Nam BAL, BAF, item CAL, CAF æquantur duobus rectis.

3. *Omnes anguli circa idem punctum æquantur quatuor rectis.* Patet ex dictis

PROPOSITIO XV.

Si duæ rectæ (BC, FL) *se secuerint, anguli ad* (A) *verticem oppositi sunt æquales.*

Nam (*per 13. h*) anguli BAL, BAF, item anguli BAL, LAC adæquant duos rectos: ergo sublato utrinque BAL, restant BAF, LAC æquales. Eodem modo ostendam BAL, FAC esse æquales.

PROPOSITIO XVIII. & XIX.

In omni triangulo angulus (A) *major est qui opponitur majori lateri* (OB:) *& è converso.*

XVIII. Ex latere majori OB aufer OF æquale lateri OA;

erit, si ducatur AF, angulus OFA (*per* 3. *Corol. p.* 32. *h*, quod ab hac non dependet) major angulo B, & (*per* 5. *h*) æqualis angulo OAF, adeoque minor angulo OAB : ergo angulus B est multò minor angulo OAB.

XIX. Conversa patet ; nam si majori angulo opponatur minus latus, major angulus opponetur minori lateri ; quod est contra præcedentem.

PROPOSITIO XX.

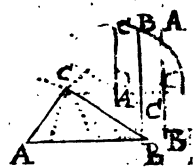
Trianguli duo quævis latera sunt reliquo majora.

Patet ex definitione lineæ rectæ.

PROPOSITIO XXII.

Ex datis tribus rectis (AB, BC, CA) *quarum duæ quævis sint reliquâ majores, triangulum constituere.*

Assumptâ quâlibet AB, centro B, intervallo datæ BC describe arcum ; & centro A, intervallo alterius datæ AC describe arcum priori occurrentem in C. Ductis AC, BC, erit ABC triangulum quæsitum. Patet *ex constr.*



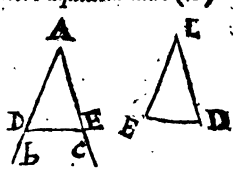
Corollarium.

Eodem modo fiet triangulum æquale cuicunque dato.

PROPOSITIO XXIII.

Ad datum punctum (L) *angulum efficere æqualem dato* (A.)

Ducatur utcunque DE Secans crura anguli A, & triangulo ADE constitue (*per* 22. *h*) æquale LDE ; eritque angulus L (*per* 8. *h*) æqualis dato (A.)



Scholium.

Quia in Operationibus Mathematicis sæpe requiruntur anguli, vel pares datis, vel certum graduum numerum continentes ; pro angulorum praxi aliqua subjungam.

1. *Ad datum punctum* (L) *statuere angulum parem dato* (A.) Centro A describe inter anguli dati latera arcum DE ; &

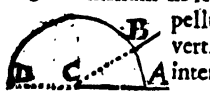
centro L, eodem radio deſcribe DF. Ex DF auſer DE æqualem alteri DE, & per E duc rectas LD, LE. Erit ELD par dato A.

2. *Anguli dati* (A) *gradus examinare.* Inter crura anguli A, centro A, deſcribe arcum DE; quem circino exceptum applica quadranti vel ſemicirculo in gradus diviſo. Notæ margini adſcriptæ quantitatem anguli indicabunt.

3. *Angulum conſtruere continentem gradus datos.* Centro quovis L deſcribatur arcus DF; ex quo ſi major ſit, auferatur DE par arcui quadrantis continenti gradus datos, & ducantur rectæ LE, LD. Erit ELD angulus quæſitus.

Nota, arcûs DE radium debere in utroque caſu æqualem eſſe ſemidiametro alicujus circuli inſtrumento inſcripti.

Si ad manum ſit ſemicirculus excavatus, vel ex materia



pellucida, ſatis erit ejus centrum ſuper C verticem anguli collocare; nam arcus AB inter crura contentus oſtendet quantitatem anguli dati ACB, ſicut & puncta B & A, per quæ à centro ductæ CB, CA efficient angulum BCA continentem gradus quæſitos.

PROPOSITIO XXIV. & XXV.

Si duo triangula (EDG, EDF) *habeant circa angulos inæquales* (EDG, EDF) *latera* (DE, DF & DE, DG) *ſibi mutuò æqualia; erit baſis* (EG) *oppoſita majori angulo* (EDG) *majori baſi* (EF) *oppoſita minori angulo* (EDF.) *& è converſo.*

XXIV. Ducatur FG. In triangulo FDG latera DF, DG ſunt (*ex ſuppoſitione.*) æqualia: ergo (*per* 5. *h*) æquales ſunt anguli DGF, DFG: ergo angulus EGF minor eſt angulo DFG, multòque minor angulo EFG: ergo (*per* 19. *h*) latus EG majus eſt latere EF.

XXV. Si anguli EDF, EDG eſſent æquales; baſes EF, EG (*per* 4. *h*) eſſent æquales. Si EDG eſſet minor, baſi EG (*per præced.*) eſſet minor. Utrumque verò eſt contra hypotheſim, nam EG ſupponitur major quàm EF. Ergo angulus EDG eſt major quàm EDF.

PROPOSITIO XXVI.

Si duo triangula (X & Z. Vide Fig. Prop. 4.) *habeant duos angulos duobus æquales* (A *ipsi* L, & B *ipsi* F) *alterum alteri, unumque latus uni lateri æquale, vel quod existit inter æquales angulos* (*ut* AB, LF) *vel quod eorum uni opponitur* (*ut* BC, FI) *omnia reliqua erunt æqualia.*

Nam (*per* 5. *Corol. p.* 32. *h.* quod ab hac non dependet) tertius angulus C æquatur tertio L. Ergo (*per Schol. p.* 4. *h.*) reliqua omnia sunt æqualia.

PROPOSITIO XXVII.

Si rectas (AB, CF) *parallelas facet recta* (GO)

1.° *Æquales sunt anguli alterni* (ALO, FOL, *item* BLO, COL.)

Duc perpendicula OR, LQ. In triangulis ROL, QLO, latus LO est commune; latera RO, LQ [*per* 36. *def. h.*] & latera RL, OQ [*per ax.* 5] sunt æqualia: ergo [*per* 8. *h.*] æquantur anguli alterni RLO, QOL; sicut & anguli ROL, QLO; quibus si addantur COR, QLB recti, adeoque æquales, æquales erunt etiam anguli alterni COL, BLO.

2.° *Externus* [GLB] *æquatur interno ad eandem partem* [LOQ.]

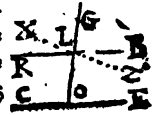
Nam [*per* 15. *h.*] GLB æquatur angulo RLO, id est [*per* 1. *partem*] angulo LOQ.

3.° *Interni ad eandem partem* [BLO, FOL, *item* RLO, COL] *æquantur duobus rectis.*

Nam [*per* 13. *h.*] BLO cum RLO, id est [*per* 1. *partem*] cum FOL, adæquat duos rectos.

PROPOSITIO XXVIII. & XXIX.

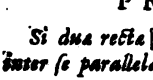
Si rectas [RB, CE *secans rectas* [GO] *facit aut angulos alternos* [RLO, EOL] *æquales; aut externum* [GLB] *æqualem interno ad eandem partem* [LOE] *aut duos internos ad eandem partem* [BLO, EOL] *æquales duobus rectis; rectæ illæ* [RB, CE] *sunt parallelæ.*



Si neges; fit XLZ parallela ad CE: ergo [*per* 27. *b*] XLZ cum GO, CE facit angulos omnes, quales descripsimus; id est [*ex suppos.*] eosdem quos cum iisdem GO, CE facit RB. Quod est absurdum: ergo RB est parallela ad CE.

PROPOSITIO XXX.

Si duæ rectæ [AB, CF] *sint parallelæ eidem* [DN,] *sunt inter se parallelæ.*



Nam si ducatur recta GO, erit [*per* 27. *b*] externus GLB æqualis interno LDN, & LDN [*per eandem*] interno DOF: ergo externus GLB [*per ax.* 1.] æquatur interno DOF seu LOF: ergo [*per* 29. *b*] parallelæ sunt AB, CF.

PROPOSITIO XXXI.

Per punctum datum ducere parallelam ad rectam datam.

Vide 2. *Corol. p.* 29. *l.* 3. quod ab hac non dependet.

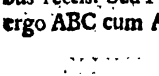
PROPOSITIO XXXII.

In omni triangulo rectilineo angulus quilibet externus (FBC) *æquatur internis oppositis* (A & C) *simul sumptis; & omnes tres anguli interni* (A, B, C) *simul conficiunt duos rectos.*

1. *Pars.* Fiat BL parallela ad AC. Angulus externus FBL (*per* 27. *b*) æquatur interno A, & alterni LBC, BCA [*per eandem*] sunt æquales. Sed FBL, LBC sunt angulus externus FBC: ergo FBC æquatur internis oppositis A & C.

2. *Pars.* Angulus ABC cum FBC (*per* 13. *b*) æquatur duobus rectis. Sed FBC (*per* 1. *partem*) æquatur angulis A & C: ergo ABC cum A & C facit duos rectos.

Corollaria.



1. *Si in triangulo quovis* (ABC) *sit aliquis angulus* [C] *rectus, reliqui* (A & B) *sunt acuti, & simul æquales uni recto.*

Nam aliter A & B cum C non conficerent duos rectos.

2. *Si angulus aliquis sit obtusus, reliqui duo sunt acuti.*

Nam aliter omnes simul excederent duos rectos. 3. *Am-*

3. *Angulus externus major est quolibet internorum oppositorum.*

Nam æquatur utrique simul.

4. *Noto uno angulo, cognoscitur Summa reliquorum: & notâ duorum Summâ, cognoscitur tertius.*

Scitur enim, quantum notis desit ad duos rectos.

5. *Cùm in uno triangulo duo anguli aut simul, aut singuli, æquales sunt duobus aut simul, aut singulis in alterô triangulo; etiam tertius tertio æqualis erit.*

Nam angulorum vel seorsim, vel simul æqualium, æqualis est differentia à duobus rectis.

6. *Perpendicularis* (AD) *est brevissima omnium, quæ ex aliquo puncto* (A) *ad rectam quamlibet* (BC) *duci possunt.*

Nam quomodocunque ducatur AD, erit angulus D rectus, adeoque B acutus: ergo (*per* 19. *h*) latus AB excedit latus AD.

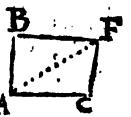
7. *Si ab uno puncto* (A) *in rectam* (EC) *cadant duæ rectæ, altera* (AB) *obliquè, altera* (AD) *perpendiculariter; hæc* (AD) *cadit versùs partes anguli acuti* (ABC.)

Nam si caderet versùs obtusum ABE, in E *v. g.* essent in triangulo AEB, angulus E (*per* 14. *def. h*) rectus, & ABE (*ex suppos.*) obtusus, quod repugnat 1. *Corol.*

PROPOSITIO. XXXIII.

Si æquales parallelas (AB, CF) *jungant rectæ* (AC, BF) *erunt hæ æquales & parallelæ.*

Ducatur AF. In triangulis BAF, CAF, æquales sunt (*per* 27. *h*) anguli alterni BAF, CFA; sicut & (*ex suppos.*) latera BA, CF, & latus AF est commune: ergo (*per* 4. *h*) æqualia sunt latera AC, BF; & anguli CAF, BFA: ergo (*per* 28. *h*) latera AC, BF sunt etiam parallela.



PROPOSITIO XXXIV.

Parallelogrammi oppositi anguli & latera sunt æqualia; ipsumque à diametro bisecatur.

1.° Ductâ diametro AF, æquales sunt (*per* 27. *h*) anguli

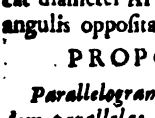
alterni BFA, CAF, item BAF, CFA: ergo totus BAC æqualis toti BFC. Similiter oſtendam angulos B & C eſſe æquales.

2.o Quia ad baſim communem AF æquales ſunt anguli BAF, CFA, & BFA, CAF; tota triangula ABF, ACF (*per ſchol. p. 4. h*) ſunt æqualia; adeoque parallelogrammum biſecat diametrer AF; & latera BF, AC, & BA, FC, æqualibus angulis oppoſita ſunt æqualia.

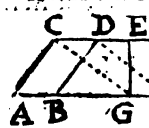
PROPOSITIO XXXV. & XXXVI.

Parallelogramma ſuper eadem vel æquali baſi & inter eaſdem parallelas poſita, ſunt æqualia.

XXXV. 1.o Sint parallelogramma (ACDB, ALQB habentia eandem baſim AB. In triangulis ALC, BQD, latera AC, DB, & AL, BQ (*per 34. h*) ſunt æqualia: & (*per 27. h*) anguli comprehenſi CAL, DBQ æquantur angulo externo DOL, adeoque (*per ax. 1.*) ſunt inter ſe æquales: ergo (*per 4. h*) tota triangula ALC, BQD ſunt æqualia: ergo ſi utrique auferas triangulum commune DOL, & utrique addas triangulum AOB, erunt (*per ax. 2.*) æqualia parallelogramma ABDC, ALQB.



XXXVI. Si habeant baſes diſtinctas ſed æquales AB, GH; ducantur GC, HD. Jam quia AB, GH ſunt (*ex ſuppoſ.*) & AB, CD (*per 34. h*) æquales; GH, CD ſunt (*per ax. 1.*) æquales. Sed (*ex ſuppoſ.*) ſunt etiam parallelæ: ergo (*per 33. h*) GC, HD ſunt æquales & parallelæ; adeoque GCDH (*per 35. def. h*) eſt parallelogrammum; cui (*per 35. h.*) æquantur parallelogramma ACDB, GEFH: ergo hæc ſunt (*per ax. 1.*) inter ſe æqualia.



Corollarium.

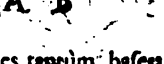
Efficere rectangulum æquale parallelogrammo dato [ACDB.]

Acceptâ baſi æquali GH, erige perpendicula GE, HF occurrentia lateri CD producto. Erunt [*per 36. h*] ACDB, GEFH æqualia.

PROPOSITIO XXXVII. & XXXVIII.

Triangula (ACB, AFB) *ſuper eadem baſi* (AB) *vel æquali & inter eaſdem parallelas conſtituta ſunt æqualia.*

XXXVII. Ducantur BL, BI parallelæ lateribus AC, AF. Parallelogramma ACLB, AFIB (*per* 35. *h*) ſunt æqualia. Sed eorum dimidia ſunt (*per.* 34. *h*) triangula ACB; AFB: ergo (*per ax.* 3.) hæc ſunt æqualia.



XXXVIII. Si triangula habeant æquales tantùm baſes; eodem modo oſtendentur eſſe dimidia parallelogrammorum iiſdem baſibus æqualibus inſiſtentium. Sed hæc (*per* 36. *h*) ſunt æqualia: ergo (*per ax.* 3.) ipſa triangula ſunt æqualia.

PROPOSITIO XLI.

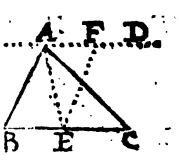
Si triangulum (AFB) *ſit in iiſdem parallelis & eâdem vel æquali baſi* (AB) *cum parallelogrammo* (LA); *erit ipſius dimidium.*

Ducatur CB triangulum AFB (*per* 37. *h*) æquatur triangulo ACB; id eſt (*per* 34. *h*) dimidio parallelogrammi AL.

PROPOSITIO XLII.

Conſtruere parallelogrammum (ABEF) *æquale triangulo dato* [ABC.]

Trianguli dati baſim BC biſeca in E; dein per anguli oppoſiti vertiecm A duc AD parallelam baſi BC, & per E duc EF parallelam alterutri lateri AB. Fritque parallelogrammum ABEF æquale triangulo dato ABC.

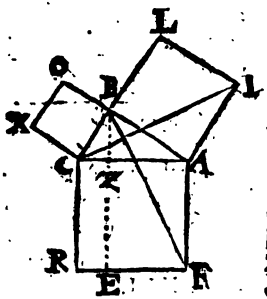


Ducatur AE; eruntque triangula ABE, AEC (*per* 38. *h*) æqualia: ergo ABE eſt dimidium trianguli ABC. Sed (*per* 41. *h*) ABE eſt dimidium parallelogrammi ABEF: ergo ABC, ABEF (*per ax.* 3.) ſunt æqualia.

PROPOSITIO XLVII.

In omni triangulo rectangulo quadratum [CAFR] *lateris* (AC) *oppoſiti angulo recto* [B] *æquatur reliquorum laterum* (AB, CB) *quadratis* [AILB, BOXC] *ſimul ſumptis.*

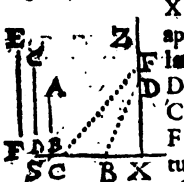
Ducantur IC & EB cum BE parallelâ ad AF. Jam ſi angulis IAB, FAC (*per* 32. *def. h.*) rectis, adeoque æqualibus addatur angulus BAC, erunt (*per ax.* 2.) anguli IAC, FAB æquales. Sed etiam latera IA, BA, & CA, FA, hos angulos continentia, ſunt (*per* 32. *def. h.*) æqualia: ergo (*per* 4. *h*) triangula IAC, FAB ſunt æqualia. Sed (*per* 41. *h*) hæc triangula ſunt dimidia rectangulorum BAIL, ZAFE: ergo [*per ax.* 3.] rectangula BAIL, ZAFE ſunt æqualia. Eodem modo oſtendam rectangula BCXO, ZERC eſſe æqualia: ergo rectangula ZAFE, ZERC, id eſt, quadratum AC æquatur rectangulis BAIL, BCXO, id eſt, quadratis AB, CB.



Corollaria.

1. *Datis pluribus quadratis, unum omnibus æquale conſtruere.*

Sint quadratorum latera AB, CD, EF. Fiat angulus rectus X habens latera infinita SX, XZ. Lateri XS applica lineam AB ab X ad B; & lineam CD lateri XZ ab X ad D: erit quadratum lateris DB æquale quadratis XB, XD, id eſt AB, CD. Rurſus EF applica lateri XZ ab X ad F; & DB lateri XS ab X ad C, erit quadratum FC æquale quadratis FX & XC, id eſt quadratis AB, CD, EF datis.

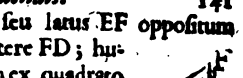


2. *Datis duobus quadratis* (EF, CD) *eorum differentiam invenire.*

Quadrati minoris latus CD applica lineæ XZ ab X ad D; & quadrati majoris latus EF ex puncto D extende ad lineam SX, & pertingat à D ad B *v.g.* erit quadratum XB differentia quadratorum XD, DB, ſeu CD, EF.

3. *Notis duobus quibuſcunque lateribus trianguli rectanguli, tertium invenire.*

1.° Si cognoscatur hypotenusa, seu latus EF oppositum angulo recto D, cum alio aliquo latere FD; hujus lateris FD quadratum subtractum ex quadrato hypotenusæ EF, relinquet quadratum lateris DE; cujus si radicem extraxeris, notum erit latus ED.



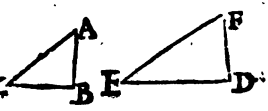
2.° Si dentur duo latera ED, FD comprehendentia angulum rectum D; eorum quadrata in summam collecta dabunt quadratum hypotenusæ EF.

4. *Si duo triangula rectangula* (ABC, FKI) *habeant hypotenusas* (AB, FK) *& latera quævis* (BC, KI) *æqualia, tertium* (AC) *æquatur tertio* (FI.)

Nam si quadratis æqualibus AB, FK, subtrahantur æqualia quadrata BC, KI; relinquentur quadrata, adeoque & latera æqualia AC, FI.

5. *Si in triangulis rectangulis* (ABC, FDE) *sint circa angulos rectos* (B & D) *latus unum* (AB) *æquale uni* (FD) *& aliud* (BC) *minus alio* (DE;) *erit unius hypotenusa* (AC) *minor alia* (EF.)

Nam quia quadrata FD, DE simul excedunt quadrata AB, CB; quadratum FE excedit quadratum AC, adeoque & recta FE excedit rectam AC.



EX LIBRO TERTIO.

DEFINITIONES.

2. Recta (AB) circulum tangit quæ illi ita occurrit in (B) ut producta illum non secet.

Subtensa arcûs (EL) est recta connectens ejus extremas (E & L.)

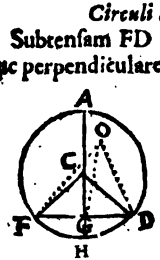
5. Segmenta circuli sunt partes, in quas eum dividit secans (EL.)

6. Angulus in segmento est (EFL) qui continetur sub rectis lineis (EF, LF) quæ ad (F) unum circumferentiæ punctum ex segmenti terminis (E, L) ducuntur.

PROPOSITIO I.

Circuli dati (FAD) *centrum invenire.*

Subtensam FD utcunque ductam bisecа in G, & per G duc perpendicularem AH, quam bisecа in C. Erit C centrum.



Si neges; sit centrum O extra rectam AH (nam nequit esse in AH, cùm ubilibet extra C dividatur inæqualiter) & ducantur OF, OG, OD. In triangulis OGF, OGD latera FG, DG (*ex constr.*) & OF, OD (*per* 18. *def. l.* 1.) sunt æqualia; & OG commune: ergo (*per* 8. *l.* 1.) anguli OGF, OGD sunt æquales, adeoque (*per* 14. *def. l.* 1.) recti; quod absurdum est, cùm AGF, AGD sint (*ex constr.*) recti.

PROPOSITIO III.

Si recta (AH) *per centrum ducta aliam* (FD) *non per centrum ductam secet bifariam, secat perpendiculariter: & è converso.*

1. *Pars.* Ducantur radii CF, CD; qui cùm æquales sint, sicut & [*ex suppos.*] latera FG, DG, & latus CG commune; triangula CFG, CDG sunt sibi mutuò æquilatera: ergo [*per* 8. *l.* 1.] anguli CGF, CGD sunt æquales: ergo [*per* 14. *def. l.* 1.] CG est perpendicularis ad FD.

2. *Pars.* In triangulis CGF, CGD, anguli ad G sunt [*ex suppos.*] recti; hypotenusæ CF, CD [*per* 18 *def. l.* 1.] æquales; & latus CG commune: ergo [*per* 4. *Corol. p.* 47. *l.* 1.] latera FG, DG sunt æqualia.

Corollaria.

1. *Datum arcum* (FHD) *bisecare.*

Per subtensam FD bisectam in G, duc perpendicularem GH; hæc bisecabit arcum datum in H.

Nam HG producta versùs A transibit (ut ostensum est *in* 1. *h*) per centrum C, ex quo si ducantur radii CF, CD; ostendetur (*ut in* 3. *h*) angulos FCG, GCD, adeoque & [*per* 28. *h.* quæ ab hac non dependet] arcus FH, HD esse æquales.

2. *Perficere arcum datum* [ABC.]

Rectas AB, BC utcunque subtensas bisecа in L & I. Per L

& I duc perpendiculares LE, IF, ſibi occurrentes in P. Erit P centrum circuli

Nam (*ex dictis*) centrum eſt tam in LE, quàm in IF; adeoque in communi puncto P.

3. *Deſcribere circulum per tria puncta data* (A, B, C) *non in directum poſita.*

Ducantur rectæ AB, BC; quibus biſectis in L & I, excitentur perpendiculares LE, IF concurrentes in P. Circulus ductus centro P, radio PC, tranſibit per puncta data.

Nam, ductis PA, PB, PC, erunt in triangulis PIB, PIC (*ex conſtr.*) latera BI, CI æqualia, PI commune, & anguli PIB, PIC recti, adeoque æquales: ergo (*per* 4. *l.* 1.) æquantur latera PB, PC; & (quod eodem modo oſtenditur) PB, PA. Ergo circulus deſcriptus per C, tranſit etiam per B & A.

PROPOSITIO V. & VI.

Circuli ſe mutuò ſecantes vel tangentes non habent idem centrum.

Patet, quòd ſi ſint concentrici, circulus cujus radius eſt major, totus cadet extra alium; nam diſtantia peripheriæ à centro ubique æquatur radio.

PROPOSITIO VII.

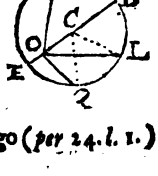
Si in circulo ex puncto (O) *quod non eſt centrum, plures rectæ* [OB, OL, OQ, OE] *in circumferentiam cadant;*

1.° *Maxima erit* (OB) *per* (C) *centrum tranſiens.*

Nam ductâ CL; patet [*ex* 20. *l.* 1.] quòd OC, CL, id eſt [quia æquales ſunt CB, CL] quòd OB major ſit quàm OL.

2.° *Reliqua diametri pars* [OE] *eſt minima.*

Nam ductâ CQ, patet quòd CO, OQ ſimul excedant CQ, id eſt CE: ergo ſublato utrinque CO, latus OQ excedit OE.



3.° *Aliarum major eſt, quæ maxima* (OB) *propior.*

Nam in triangulis COQ, CLO, latera LC, CO æquantur lateribus QC, CO: angulus verò LCO major eſt quàm QCO: ergo (*per* 24. *l.* 1.) baſis OL excedit baſim OQ.

4.° *Ex O quod non est centrum, non plures quàm duæ ad circumferentiam duci possunt æquales.*

Patet; nam si ducerentur tres æquales OL, OF, OQ, essent ad eandem partem duæ æquales OL, OQ; quod repugnat 3. *parti.*

PROPOSITIO IX.

Si ab aliquo intra circulum puncto plures quàm duæ ad ambitum duci possunt æquales, illud punctum est centrum.

Patet *ex* 4. *parte præcedentis.*

PROPOSITIO X.

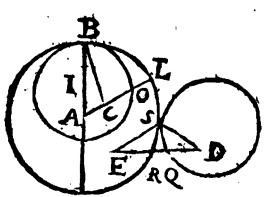
Circuli in duobus tantùm punctis se mutuò secant.

Si se mutuò secuerint in tribus; rectæ ex centro utriusvis circuli ductæ ad illa tria puncta, erunt (*per* 18. *def. l.* 1.) æquales: ergo (*per* 9. *h*) centrum illud est utrique circulo commune: ergo circuli concentrici se mutuò secant; quod est contra quintam *h*.

PROPOSITIO XI. & XII.

Si duo circuli se quomodocunque tangant, recta eorum centra connectens per contactum transit.

XI. 1.° Se tangant interiùs in B. Si AB ducta ex A centro circuli BL ad contactum B non transeat per centrum circuli BO, erit centrum illud alicubi extra lineam AB, *v. g.* in C. Ducantur per C rectæ AL, BC. Quia C est centrum, CB, CO sunt æquales; & additâ utrique AC, erunt AC, CB æquales AO: sed AC, CB sunt majores (*per* 20. *l.* 1.) quàm AB, hoc est (*per* 18. *def. l.* 1.) quàm AL: ergo AO major est quàm AL; quod est absurdum.



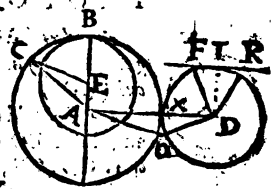
XII. 2.° Se tangant exteriùs in S, & si fieri potest, ita ponantur centra in E & D, ut recta ED non transeat per contactum S. Quia E & D sunt centra, radii ES, ER, item DQ, DS sunt æquales: ergo tota ED major est quàm ES & DS, quod repugnat vigesimæ *l. primi.*

PRO-

PROPOSITIO XIII.

Circuli se mutuò & lineam rectam punctualiter tangunt.

1.° Si fieri potest, tangant se interiùs in duobus punctis B & C. Recta AE ducta per centra A & E (*per* 11. *h*) transit per contactum, putà per B: ducantur etiam AC, EC. Quia E est centrum, EB, EC sunt æquales: ergo additâ utrique AE, rectæ AE, EC æquales sunt radio AB, seu AC, quod est contra vigesimam *l.* 1.



2.° Tangant se exteriùs in X & O. Erunt radii AX, DX, seu (*per* 12. *h*) recta AD, æquales radiis AO, DO, quod est contra vigesimam *l.* 1.

3.° Recta FR tangat circulum in F & R; erunt radii DF, DR æquales: ergo (*per* 5. *l.* 1.) anguli DFR, DRF sunt æquales; adeoque acuti: ergo DI ducta perpendiculariter à centro D ad rectam FR (*per* 6. & 7. *Corol. p.* 32. *l.* 1.) cadet inter F & R; eritque minor radio DR: ergo punctum I est intra circulum: ergo FR secat circulum.

PROPOSITIO XVI.

Recta (AB) *quæ perpendicularis est ad diametri extremitatem* (A), *tota cadit extra circulum.*

Ad rectæ AB quodvis punctum B ducatur à centro recta CB. Quia angulus CAB ponitur rectus, CBA [*per* 1. *Corol. p.* 32. *l.* 1.] est acutus: ergo (*per* 19. *l.* 1.) CB major quàm CA: ergo punctum B, & (ob eandem rationem) omnia puncta lineæ AB, cadunt extra circulum.

PROPOSITIO XVII.

Ex puncto dato rectam ducere, quæ tangat circulum datum. Vide *Corol. p.* 31. *h.*

PROPOSITIO XVIII.

Recta [CA] *quæ ducitur ex centro ad contactum* [A] *est tangenti* (AB) *perpendicularis.*

Si neges; sit CB perpendicularis; erit angulus B rectus, adeoque [*per* 1. *Corol. p.* 32. *l.* 1.] A acutus: ergo (*per* 19. *l.* 1.) CB minor est quàm CA, id est (*per* 18. *def. l.* 1.) quàm CD, quod est absurdum.

PROPOSITIO XX.

Angulus (ACD) *ad centrum dimidius est anguli* (AFD) *ad peripheriam insistentibus in arcu* (AD.)

1.o Coincidant AC, AF, erit (*per* 32. *l.* 1.) angulus ACD æqualis angulis D & F; id est, quia CD, CF, adeoque (*per* 5. *l.* 1.) anguli D & F sunt æquales; duplus anguli F.

2.o Si AG, DC cadant inter AF, DF, ducatur diameter FE; & erit (*ut prius*) ECA duplus anguli EFA, & ECD duplus anguli EFD: ergo totus ACD est duplus totius AFD.

3.o Si unum latus AF secet aliud DC, ducatur diameter FE; & erit (*ut prius*) ECD duplus anguli EFD, & ECA duplus anguli EFA: ergo sublatis ECA, EFA, restans ACD duplus anguli AFD.

PROPOSITIO XXI.

Anguli (ADB) *ad peripheriam, qui in circulo insistunt eidem arcui, vel sunt in eodem segmento, sunt æquales.*

1.o Si segmentum ADB sit majus semicirculo, angulus ad centrum ACB (*per* 20. *b*) est duplus singulorum ADB: ergo (*per ax.* 3.) anguli ADB sunt inter se æquales.

2.o Si segmentum ADB minus sit semicirculo, vel æquale; in triangulis DAT, DBT, anguli ad T (*per* 15. *l.* 1.) & anguli A & B (*per* 1. *partem b*) sunt æquales: ergo [*per* 5. *Corol. p.* 32. *l.* 1.] angulus tertius D æquatur tertio D.

PROPOSITIO XXIII.

Quadrilateri (ABCD) circulo inscripti quinis duo anguli oppositi (A & C, B & D) simul faciunt duos rectos.

Ducantur AC, BD. Angulus ABC cum BCA, & BAC (*per* 32. *l.* 1.) facit duos rectos: sed (*per* 21. *h*) anguli BCA, BDA, & BAC, BDC sunt æquales: ergo ABC cum BDA, BDC, id est cum ADC facit duos rectos.

PROPOSITIO XXV.

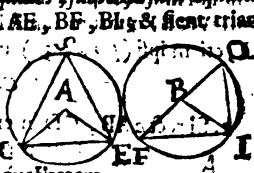
Datum arcum perficere.

Vide 2. *Cheol. p.* 3. *h.*

PROPOSITIO XXVI. & XXVII.

In æqualibus circulis recta æquales (CE, FI) subtendunt arcus æquales; & si arcus sint æquales, subtensæ sunt æquales.

XXVI. Ducantur radii AC, AE, BF, BI; & fient triangula ACE, BFI sibi mutuò æquilatera, adeoque (*per* 8. *l.* 1.) anguli A & B æquales: ergo cruta BF, BI impositæ cruribus AC, AE, ipsis congruent; fient & arcus IF arcui CE ob circulorum æqualitatem.



XXVII. Quia arcus CE, FI ponuntur æquales, sibi mutuò impositi congruent, & puncta F, I cadent in puncta C, E: ergo subtensæ sibi mutuò congruent; adeoque [*per ax.* 4.] erunt æquales.

Corollarium.

Inter arcus (CE, IO) semicirculo minores, ille (CE) est major, cujus major est subtensa.

Nam si ducantur radii AC, AE, BI, BO; erunt (*per* 18. *def. l.* 1.) in triangulis CAE, IBO, latera AC, BI, & AE, BO æqualia, & basis CE (*ex suppos.*) major basi IO: ergo (*per* 25. *l.* 1.) angulus CAE major est angulo IBO: ergo si latus BI impositum lateri AE, latus BO cadet inter C & E, & arcus IO non pertinget ad C ad E, adeoque arcus IO minor est arcu CE.

PROPOSITIO XXVIII. & XXIX.

In circulis æqualibus si anguli, sive ad centra, sive ad ambitum sint æquales; arcus, quibus insistunt, sunt æquales: & è converso.

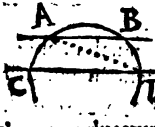
XXVIII. 1. *Pars.* Quia in triangulis ACE, BFI, latera AC, BF, & AE, BI, & anguli contenti A & B sunt (*ex suppos.*) æquales; æquantur (*per* 4. *l.* 1.) latera CE, FI; adeoque (*per* 27. *h*) etiam arcus CE, FI.

2. *Pars.* Anguli CAE, FBI sunt (*per* 20. *h*) dupli angulorum CSE, FOI: ergo (*per ax.* 3.) si anguli CSE, FOI sint æquales, æquantur anguli CAE, FBI; adeoque (*per* 1. *partem*) etiam arcus CE, FI.

XXIX. Quia arcus CE, FI ponuntur æquales, æquantur (*per* 27. *h*) subtensæ CE, FI; adeoque triangula ACE, BFI sunt sibi mutuò æquilatera; ergo (*per* 8. *l.* 1.) æquales sunt anguli A & B; quorum dimidia cùm sint (*per* 20. *h*) anguli S & O, etiam ipsi (*per ax.* 3.) sunt æquales.

Corollaria.

1. Si rectæ AB, CD intercipiant arcus æquales AC, BD; sunt parallelæ. Nam ductâ AD, anguli alterni BAD, CDA [*per* 29. *h*] sunt æquales: ergo [*per* 28. *l.* 1.] AB, CD sunt parallelæ.



2. Si per datum punctum D oporteat parallelam ducere ad rectam datam AB; quovis centro ducatur circulus transiens per D, & secans utcunque datam AB in B & A, arcui BD accipe æqualem AC, recta CD erit parallela datæ AB.

PROPOSITIO XXX.

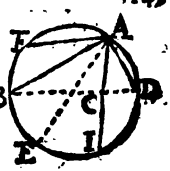
Datum arcum bisecare.

Vide 1. *Corol. p.* 3. *h.*

PROPOSITIO XXXI.

Angulus (BAD) in semicirculo est rectus; in segmento majore est minor recto; in minore est major recto.

1. *Pars.* Ducantur diametri AE, BD; eritque (*per* 30. *h*)



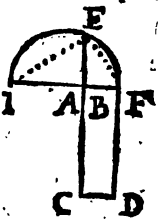
angulus BCE duplus anguli BAE, & angulus ECD duplus anguli EAD. Sed BCE, ECD simul [*per* 13. *l.* 1.] æquantur duobus rectis: ergo BAE, EAD simul, id est BAD æquatur uni recto.

2. & 3. *Pars.* Patet quòd, quomodocunque ducantur IA, & FA, erunt in segmento majore angulus IAD minor, & in segmento minore, angulus FAD major recto BAD.

Corollaria.

1. *Quadratum exhibere æquale cuicunque Figuræ rectilineæ datæ.*

1.° Proponatur rectangulum BCDF. Latus minus FB productum in I, donec æquetur majori BC, bisecta in A; & centro A per I & F describe semicirculum, cui in E occurrat CB productum. Demum duc rectas IE, EF; eritque angulus IEF rectus, & quadratum EF æquale rectangulo BCDF. Demonstratur ut Prop. 47. *l.* 1.



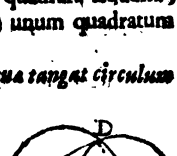
2.° Si detur parallelogrammum obliquangulum fiat (*per Corol. p.* 36. *l.* 1.) rectangulum ei æquale, & (*per* 1. *partem*) quadratum æquale huic rectangulo.

3.° Si detur triangulum, fiat (*per* 42. *l.* 1.) ei æquale parallelogrammum, & (*per* 1. & 2. *partem*) huic æquale quadratum.

4.° Si proponatur alia quævis Figura, resolvatur in triangula; quibus (*per* 3. *partem*) fiant totidem quadrata æqualia; & his omnibus (*per* 1. *Corol. p.* 47. *l.* 1.) unum quadratum æquale.

2. *Ex dato puncto* (A) *rectam ducere, quæ tangat circulum datum.*

Ab A ad circuli dati centrum C ducatur recta AC; quâ bisectâ in B, centro B, radio BA, describatur semicirculus occurrens circulo dato in D. Recta AD tanget circulum datum.



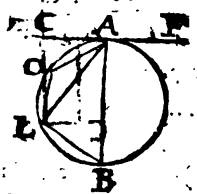
Nam, ductâ CD, angulus ADC in semicirculo [*per* 31. *h*] est rectus: ergo [*per* 17. *h*] AD tangit circulum.

3. *Ad lineæ datæ extremum punctum* [B] *ducere perpendiculum* (AB.)

Accepto utcunque centro F extra datam BC, radio FB describatur circulus ABC secans datam alicubi in C; ductâque diametro CA, jungantur A & B. Erit AB perpendicularis ad BC in B; nam (*per* 31. *h*) angulus ABC est rectus.

PROPOSITIO XXXII.

Si recta (CF) *circulum tangat, & alia* (AB, *vel* AL, *vel* AO) *ex* (A) *contactu ductâ eundem secet; erit angulus à tangente & secante factus, par angulo qui sit in segmento alterno: id est, æquales erunt anguli* FAB, ALB, & CAL, ABL, & FAL, AOL.



1.° Si secans AB transeat per centrum, recti sunt anguli, FAB (*per* 18. *h*) & ALB (*per* 31. *h*) adeoque æquales.

2.° Si secans AL per centrum non transeat; ducatur per centrum recta AB, & jungantur L & B. Quia angulus ALB (*per* 31. *h*) est rectus, anguli LBA, LAB (*per* 1. *Corol. p.* 32. *l.* 1.) simul efficiunt unum rectum. Sed (*per* 18. *h*) CAL, LAB simul, seu CAB æquatur uni recto: ergo ablato communi LAB, restat CAL æqualis angulo LBA in segmento alterno.

3.° Quadrilateri AOLB anguli oppositi O & B (*per* 22. *h*) simul adæquant duos rectos. Sed LAC, LAF (*per* 13. *l.* 1.) simul constituunt duos rectos: ergo subductis LAC, LBA (*per* 2. *partem*) æqualibus, restat LAF æqualis angulo LOA in segmento alterno.

PROPOSITIO XXXV.

Si duæ rectæ se in circulo secuerint, rectangulum sub segmentis unius, æquale est rectangulo sub segmentis alterius.

Vide *Corol. p.* 16. *l.* 6.

PROPOSITIO XXXVI.

Si ex puncto extra circulum una recta tangat, alia secet circulum; erit quadratum tangentis æquale rectangulo sub tota secante & parte inter punctum & circulum interjectâ.

Vide 1. *Corol. p.* 17. *l.* 6. ubi etiam duo hujus Propositionis Corollaria demonstrantur.

EX LIBRO QUINTO.

Quia Libri hujus Theoremata, non solùm lineis, sed omni omninò quantitati conveniunt, ad terminos exprimendos Alphabeti literas adhibemus, quæ Propositionum universalitatem melius indicant. Brevitatis etiam & claritatis causâ utimur notis quibusdam, apud Geometras receptissimis; quas ne Tyronem morentur, breviter explico.

1.° Proportio sic exprimitur A : B :: C : D. Lege A est ad B, ut C ad D.

2.° = Significat æqualitatem. Ut A = , id est quantitas A æqualis est quantitati B.

3.° + Est signum Additionis. Sic A + B significat A plus B, seu A cum B.

4.° − Denotat subtractionem. Sic A − B significat A minùs B, seu A mulctatam quantitatem B.

Propositiones aliquæ hujus Libri sunt per se ita manifestæ, ut probatione non indigeant; quam proinde omisimus, ne (quod interdum contingit) res per se notæ demonstrando fierent obscuræ.

DEFINITIONES.

3. *Ratio* est duarum ejusdem generis magnitudinum mutua secundùm quantitatem habitudo. Quantitas quæ comparatur, dicitur *Antecedens*: illa, cui comparatur *Consequens*.

5 Rationes sunt *similes*, *Æquales*, *Eadem*, quarum Antecedentia eandem habent magnitudinem respectu suorum Consequentium; ut si A sit semissis vel duplum B, C semissis vel duplum D.

Proportio est rationum similitudo, æqualitas, vel identitas.

8. Datur *Proportio continua*, quando terminus qui est Con-

sequens respectu unius Antecedentis, est Antecedens respectu alterius Consequentis; seu quando medii termini bis sumuntur; ut A: B :: B: C. Datur *Proportio discreta*, cùm nullus terminus bis accipitur; ut A: B :: C: D.

10. Si dentur plures termini (A, B, C, D &c.) continuè proportionales; Primus (A) habet ad Tertium (C) rationem *Duplicatam*, & ad Quartum (D) rationem *Triplicatam* ejus, quam idem Primus (A) habet ad Secundum (B.)

Sit *ex. gr.* 2: 4:: 4: 8; & 4: 8:: 8: 16. erit ratio primi 2 ad tertium 8 duplicata rationis primi 2 ad secundum 4. & ratio primi 2 ad quartum 16 triplicata rationis primi 2 ad secundum 4. Id est, quia secundus terminus 4 est duplum primi 2; tertius 8 erit duplum secundi 4, adeoque dupliciter duplum primi 2: & quartus 16, quia est duplum tertii 8, erit quadrupliciter duplum primi 2. Et sic deinceps.

Similiter sit 1: 3:: 3: 9, & 3: 9:: 9: 27. Quia secundus 3 est triplum primi 1, tertius 9 est triplum secundi 3; adeoque tripliciter triplum primi 1: & quartus 27, quia triplum est tertii 9, est nonies triplum primi 1.

11. Termini sunt *Homologi*, qui ad sua Antecedentia vel Consequentia eandem habent rationem.

PROPOSITIO VII.

Si dua quantitates (A, B) *fuerint æquales, eandem habebunt rationem ad tertiam quampiam* (Z) *& è converso.*

Hoc est, erit A: Z:: B: Z. & è converso Z: A:: Z: B.

PROPOSITIO VIII.

Si dua [A & B] *fuerint inæquales; major* (A) *ad tertiam* [Z] *habebit majorem rationem, quàm minor* (B) *ad eandem* (Z).

Et è converso tertia quævis (Z) *minorem habet rationem ad quantitatem majorem* (A) *quàm ad minorem* (B.)

PROPOSITIO IX.

Si sit A: Z:: B: Z, *vel* Z: A:: Z: B; A & B *sunt æquales.*

PROPOSITIO X.

Si C *ad* Z *majorem rationem habet quàm* F *ad* Z, *erit* C *major*

major quàm F; *& si* Z *ad* F *majorem rationem habet quàm* Z *ad* C, *erit* F *minor quàm* C.

PROPOSITIO XI.

Rationes eidem tertiæ æquales, sunt æquales inter se.

Hoc est, si sit A: B:: X: Z, & C: D:: X: Z; erit A: B:: C: D.

PROPOSITIO XII.

Si A, B, C, *singulæ rationem eandem habuerint ad singulas* F, I, L; *erit ut* A *ad* F, *ita complexum omnium* A, B, C, *ad complexum omnium* F, I, L.

Id est, si singulæ A, B, C sint *v. g.* triplæ singularum F, I, L erunt A, B, C simul sumptæ triplæ simul sumptarum F, I, L.

PROPOSITIO XV.

Partes similes sunt inter se, ut sua tota.

Patet; nam illæ dicuntur partes *similes*, quæ eodem modo continentur in suis totis; id est, quæ sunt totorum dimidia, tertiæ, quartæ &c.

PROPOSITIO XVI.

Si sit A: B:: C: D; *erit* (*permutando, id est comparando primum cum tertio, & secundum cum quarto*) A: C:: B: D.

Sint enim B & D *v. g.* minores quàm A & C; erit (*per* 15. *h*) ratio totorum A & C, eadem quæ partium similium B & D.

Corollarium.

Si sit A: B:: C: D; *erit* (*invertendo, id est comparando secundum cum primo, & quartum cum tertio*) B: A:: D: C.

Patet; nam si A continet B, sicut C continet D; B continetur in A, sicut D in C.

PROPOSITIO XVII.

Si sit A: B:: C: D; *erit* (*dividendo*) A − B: B:: C − D: D.

Nam A & C mulctatæ similibus partibus, retinent (*ut patet*) eandem rationem ad se invicem: ergo (*per* 15. *h*) & ad suas partes similes B & D.

PROPOSITIO XVIII.

Si sit A: B:: C: D; *erit* (*componendo*) A + B: B:: C + D: D.

Nam totæ A & C dum augentur partibus similibus [*per* 11. *h*] servant eandem proportionem inter se: ergo (*per* 15. *h*) & ad suas partes similes B & D.

PROPOSITIO XIX.

Si sit totum A *ad ablatum* B, *ut totum* C *ad ablatum* D; *erit* A: C:: A —B: C —D.

Nam si totis A & C auferantur partes similes B & D relinquentur partes similes A —B & C —D, quæ (*per* 15. *h*) sunt inter se, ut suæ totæ A & C.

Corollarium.

Si totum A *sit ad suî partem* B, *ut totum* C *ad suî partem* D; *erit* (*per conversionem rationis*) A: A —B:: C: C —D.

Nam [*per* 19. *h*] A: C:: A —B: C —D. Ergo (*permutando*) A: A —B:: C: C —D.

PROPOSITIO XXII.

Si sit A: B:: C: D, & B: E:: D: F; *erit* (*ex æquo ordinata*) A: E:: C: F.

Nam quia ponitur A: B:: C: D; erit (*permutando*) A: C:: B: D. Et quia ponitur B: E:: D: F; erit (*permutando*) B: D:: E: F. Ergo (*per* 11. *h*) A: C:: E: F, & (*permutando*) A: E:: C: F.

Corollarium.

Si sit A: B:: C: D, & E: F:: I: L; *erit* (*multiplicatis inter se utriusque Analogiæ terminis*) AE: BF:: CI: DL.

Hoc est rectangulum sub A & E erit ad rectang. sub B & F: ut rectang. sub C & I, ad rectang. sub D & L.

Nam (*per* 1. *l.* 6. quæ ab hac non dependet) BE: BF:: E: F, & DI: DL:: I: L. Sed [*ex suppos.*] E: F:: I: L. Ergo (*per* 11. *h*) BE: BF:: DI: DL.

Rursus [*per* 1. *l.* 6] AE: BE:: A: B, & CI: DI:: C: D. Sed (*ex suppos.*) A: B:: C: D. Ergo (*per* 11. *h*) AE: BE:: CI: DI.

Atqui (*ex dictis*) BE: BF:: DI: DL. Ergo (*ex æquo ordinata*) AE: BF:: CI: DL.

PROPOSITIO XXIII.

Si sit 1A: 2B:: 1D: 2F, & 2B: 3E:: 3C: 1D; *erit* (*ex æquo perturbata*) 1A: 3E:: 3C: 2F.

Fiat ut B ad E, ſic F ad aliquam aliam O.

Jam quia eſt [*ex ſuppoſ.*] A: B:: D: F, & (*ex conſtr.*) B: E:: F: O; erit (*per 22. h*) A: E:: D: O.

Rurſus quia ſunt (*ex ſuppoſ.*) B: E:: C: D, & [*ex conſtr.*] B: E:: F: O; erit [*per 11. h*] C: D:: F: O, & (*permutando*) C: F:: D: O.

Sed eſt (*ex diſtis*) A: E:: D: O. Ergo (*per 11. h*) A: E:: C: F.

PROPOSITIO XXIV.

Si ſit A: B:: C: D, & E: B:: F: D; *erit* A + E: B:: C + F: D.

Quia eſt [*ex ſuppoſ.*] E: B:: F: D; erit (*invertendo*) B: E:: D: F. Cùm igitur ſit (*ex ſuppoſ.*) A: B:: C: D, & (*ex diſtis*) B: E:: D: F; erit (*per 22. h*) A: E:: C: F. Ergo (*componendo*) A + E: E:: C + F: F. Sed etiam [*ex ſuppoſ.*] E: B:: F: D. Ergo [*per 22. h*] A + E: B:: C + F: D.

EX LIBRO SEXTO.

DEFINITIONES.

1. Figuræ dicuntur *ſimiles*, quæ ſibi mutuò ſunt æquiangulæ, & latera æqualibus angulis oppoſita habent proportionalia.

2. Figuræ ſunt *reciprocæ*, cùm in utraque antecedentes & conſequentes rationum termini fuerint. Sic in parallelogrammis X, Z, (*Vide Fig. Prop. 14.*) Antecedens AC: Conſequens OB:: Antecedens FO: Conſequens CL.

3. *Altitudo* figuræ eſt longitudo perpendiculi DC ab ejus vertice ad baſim ducti. Unde triangula æquè alta ſunt inter eaſdem parallelas: & è contrà.

4. Arcus ſunt *ſimiles* qui ad ſuas circumferentias eandem habent rationem.

PROPOSITIO I.

Triangula (ABC, MHO) *& Parallelogramma* [AB, MH] *habentia eandem altitudinem, ſunt inter ſe ut baſes.*

1. *Pars.* Sit MO tertia pars baſeos AC. Dividatur AC in tres partes æquales AD, DE, EC, & ducantur lineæ BD, BE.

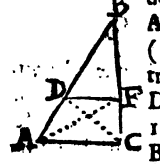
Quia triangula ABD, DBE, EBC habent [*ex suppos.*] eandem altitudinem, & (*ex constr.*) baſes æquales, ſunt (*per 38. l. 1.*) inter ſe æqualia; & (ob eandem rationem) eorum quodvis æquatur triangulo MHO: ergo MHO eſt tertia pars totius ABC. Sed (*ex suppos.*) baſis MO eſt tertia pars baſeos AC: ergo triangula ABC, MHO ſunt inter ſe ut baſes AC, MO.

2. *Pars.* Cùm Parallelogramma AB, MH ſint (*per 41. l. 1.*) dupla triangulorum ABC, MHO; erunt etiam inter ſe ut baſes.

PROPOSITIO II.

Si in triangulo (ABC) *ducatur aliqua* (DF) *parallela ad unum latus* (AC) *ſecabit alia latera* (AB, BC) *proportionaliter; id eſt, erit* BD: DA:: BF: FC.

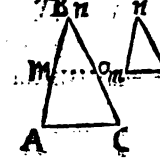
Ducantur AF, CD. Triangula DAF, DCF, habentia eandem baſim DF, & inter eaſdem parallelas DF, AC, ſunt (*per 37. l. 1.*) inter ſe æqualia: ergo (*per 7. l. 5.*) eandem rationem habent ad triangulum DBF. Sed (*per 1. b*) DAF: BDF:: DA: BD. & DCF: DBF:: FC: BF. Ergo (*per 11. l. 5.*) DA: BD:: FC: BF. & (*invertendo*) BD: DA:: BF: FC.



PROPOSITIO IV.

Triangula ſibi mutuò aquiangula ſunt ſimilia.

In triangulis ABC, MNO ſint æquales anguli B & N, A & M, C & O. Si angulus N ponatur ſuper æqualem B, latera NM, NO incident in latera BA, BC; & anguli NOM & BCA erunt (*ex suppoſ.*) æquales: ergo (*per 29. l. 1.*) AC & MO ſunt parallelæ: ergo (*per 2. b*) AM: MN:: CO: ON. Et (*componendo*) AN ſeu AB: MN:: CN ſeu CB: ON. Et (*permutando*) AB: CB:: MN: NO.



Corollaria.

Si in triangulo aliquo (ABC) *ducatur* (MO) *parallela ad*

latus aliquod (AC;) *sient duo triangula similia* (BMO, BAC.)

Nam angulus B est communis: & externi BMO, BOM (*per* 27. *l.* 1.) æquantur internis BAC, BCA: ergo triangula sunt æquiangula, adeoque similia.

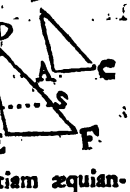
2. *Si duo triangula* (ABC, MNO) *habeant quosvis duos angulos* (A & M, B & N) *æquales sunt similia.*

Nàm (*per* 5. *Corol. p.* 32. *l.* 1.) tertius angulus C æquatur tertio O: ergo &c.

PROPOSITIO V.

Si triangula (ABC, DEF) *habeant omnia latera sibi mutuò proportionalia, sunt sibi mutuò æquiangula.*

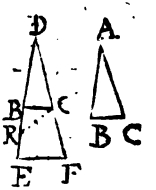
Sit *v. gr.* DE: BA:: EF: AC. & sint latera DE, DF majora quàm BA, BC. ex DE accipe DO æquale lateri BA, & per O duc OS parallelam lateri EF. erit (*per* 1. *Corol. p.* 4. *h*) ED: OD seu AB:: EF: OS. Sed (*ex suppos.*) ED: AB seu OD:: EF: AC. Ergo æqualia sunt OS & AC. Similiter ostendam DS & BC esse æqualia: ergo triangula ODS, ABC sunt sibi mutuò æquilatera; adeoque (*per* 8. *l.* 1.) etiam æquiangula. Sed ODS, EDF (*ut patet ex* 27. *l.* 1.) sunt æquiangula: ergo EDF, ABC sunt æquiangula.



PROPOSITIO VI.

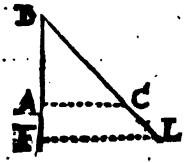
Si triangula (EDF, BAC) *habeant circa angulos æquales* (D & A) *latera* (ED, FD, AB, AC) *proportionalia, id est,* ED: AB:: FD: AC vel FD: AC:: ED: AB; *triangula sunt similia.*

Si angulus A ponatur super æqualem D, AB cadet in DE, & AC in DF. Jam si BC, EF sint parallelæ, triangula (*per* 1. *Corol. p.* 4. *h*) sunt similia. Si neges esse parallelas, ducatur CR Parallela ad EF: ergo (*per* 1. *Corol. p.* 4. *h*) FD: CD seu CA:: ED: RD. Sed (*ex suppos.*) FD: CA:: ED: BA seu BD. Ergo æqualia sunt RD, BD, totum & pars, quod est absurdum.



PROPOSITIO IX.

Rectam (BF) *secare secundum proportionem datam* (LC ad CB.)



Datis BF, BL, constitutis ad angulum quemvis FBL, ducatur FL, cique parallela AC: erit (*per* 1. *Corol. p.* 4. *h*) FA: AB:: LC: CB.

PROPOSITIO XI.

Rectis (AB, BC) *invenire tertiam proportionalem.*

Ex AB productâ accipe BF æqualem datæ BC, & per A & F duc parallelas AC, FL. erit (*per* 1. *Corol. p.* 4. *h*) AB: BC:: BF seu BC: BL.

PROPOSITIO XII.

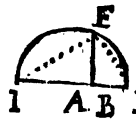
Datis tribus [BA, BC, BF] *invenire quartam proportionalem.*

Ductis AC, FL parallelis, erit AB: BC:: BF: BL.

PROPOSITIO XIII.

Datis duabus rectis [IB, BF] *invenire mediam proportionalem.*

Totâ compositâ IF bisectâ in A, centro A, describe circulum per I & F, & ex B erige perpendicularem BE occurrentem circulo in E. erit IB: BE:: BE: BF.



Ducantur IE, EF. In triangulis IEF, IEB, recti sunt, adeoque æquales, angulus E [*per* 31. *l.* 3.] & angulus B (*per constr.*) & angulus I communis: ergo (*per* 9. *Corol. p.* 32. *l.* 1.) triangula IEF, IBE sunt æquiangula. Similiter in triangulis IEF, FBE, anguli E & B sunt recti, & F communis: ergo triangula IEF, FBE sunt æquiangula: ergo (*per ex.* 1.) triangula IBE, FBE sunt æquiangula, adeoque (*per* 4. *h*) similia: ergo IB: BE:: BE: BF.

Corollaria.

1. In triangulo rectangulo perpendicularis (EB) secat

triangulum (IEF) in partes (IEB, EFB) rect. & inter se similes.

2. EB est media proportionalis inter IB, BF; & EF inter IF, BF; & IE inter IF, IB.

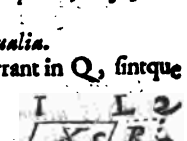
3. Hinc habetur alius modus facilis inveniendi mediam proportionalem inter duas datas, *v. g.* IF & IB. Bisectâ majore IF, per I & F duc semicirculum; & IB posito super IF ex I in B, à puncto B excita perpendiculum BE ad peripheriam terminatum. Ducatur denique IE, & erit (ut ostensum est) IB: IE:: IE: IF.

PROPOSITIO XIV. & XV.

Æqualia Parallelogramma (AILC, FOBS) *& triangula* (FAC, OLB) *habentia unum angulum* (L) *uni* (O) *æquali, habent latera circa æquales angulos reciproca; id est*, AC: OB:: FO: CL.

Et quæ latera habent reciproca, sunt æqualia.

XIV. 1. *Pars.* SB, IL productæ concurrant in Q, sintque tria parallelogramma X, Z, R. *Per* 1. *h.* X: R:: AC: OB; & Z: R:: FO: CL. Sed quia sunt (*ex suppos.*) æqualia X & Z, est (*per* 7. *l.* 5.) X: R:: Z: R; ergo (*per* 11. *l.* 5.) AC: OB:: FO: CL.



2. *Pars.* Est (*per* 1. *h*) X: R:: AC: OB; & Z: R:: FO: CL. Sed (*ex suppos.*) AC: OB:: FO: CL: ergo (*per* 11. *l.* 5.) X: R:: Z: R. Ergo (*per* 9. *l.* 5.) X & Z sunt æqualia.

XV. Ducatur AL, & similiter demonstres in triangulis FAC, OLB, esse AC: OB:: OL: FC.

PROPOSITIO XVI.

Si quatuor recta (AC, OB, FO, LC) *sint proportionalia, rectangulum* (X) *sub extremis* (AC, LC) *æquatur rectangulo* (Z) *sub mediis* (OB, FO.)

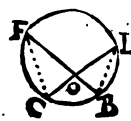
Patet (*ex.* 14. *h*;) nam habebunt latera reciproca; est enim (*ex suppos.*) AC: OB:: FO: LC.

Corollarium.

Si in circulo se secuerint duæ rectæ (CL, BF) *rectangulum*

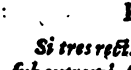
(COL) *sub segmentis unius æquatur rectangulo* (BOF) *sub segmentis alterius.*

Ducantur CF, BL. Angulus F (*per* 21. *l.* 3.) æquatur angulo L insistenti eidem arcui CB, & anguli ad O (*per* 15. *l.* 1.) sunt æquales: ergo (*per* 2. *Corol. p.* 4. *h*) CO: OF:: BO: OL. Ergo (*per* 16. *h*) rectangulum sub CO, OL æquatur rectangulo sub OF, BO.



PROPOSITIO XVII.

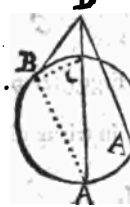
Si tres rectæ (AB, FL BC) *sint proportionales, rectangulum sub extremis* (AB, BC) *æquatur quadrato mediæ* (FL.)



Accipe LO æqualem FL, & erit (*per* 7. *l.* 5.) AB: FL:: LO: BC. Ergo [*per* 16. *h*] rectangulum sub AB, BC æquale est rectangulo sub FL, LO, id est [*per def.* 32. *l.* 1.] quadrato FL.

Corollaria.

Si ex puncto (D) *extra circulum posito ducatur tangens* (DB) *& utcunque secans* [DA] *erit quadratum tangentis* (DB) *æquale rectangulo* [ADC] *sub tota secante* (DA) *& parte* [DC] *extra circulum.*



Ducantur BC, BA. In triangulis ADB, BCD, angulus D communis est, & angulus DBC (*per* 32. *l.* 3.) æqualis angulo BAD in segmento alterno: ergo (*per* 2. *Corol. p.* 4. *h*) DA: DB:: DB: DC: ergo (*per* 17. *h*) rectangulum ADC æquatur quadrato DB.

2. *Duæ rectæ quæ ex eodem puncto* [D] *circulum tangunt, sunt æquales.*

Nam earum quadrata æquantur singula eidem rectangulo ADC.

3. *Quotcunque circulum secent ex eodem puncto* [D] *omnia rectangula erunt æqualia.*

Nam singula æquantur quadrato DB.

PROPOSITIO XIX.

Triangulorum similium [ABC, FLI] *proportio est duplicata proportionis laterum* (AC, FI) *æqualibus angulis* (B, L) *subtensorum.*

Hoc

Hoc eſt ſi fiat ut AC ad FI, ſic FI ad AQ; erit ABC: FLI:: AC: AQ.

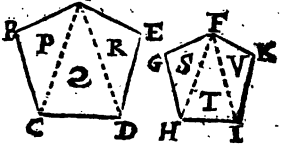
Quia triangula ponuntur ſimilia, eſt (*per 4. h*) AB: IL:: AC: IF. Id eſt (*per conſtr.*) ut IF ad AQ. Ergo in triangulis QBA, FLI latera circa æquales angulos A & I ſunt reciproca. Ergo (*per 15. h*) QBA, FLI ſunt æqualia: ſed (*per 1. h*) ABC: QBA:: AC: AQ. Ergo (*per 7. l. 5.*) ABC: FLI:: AC: AQ.

PROPOSITIO XX.

PARS PRIMA.

Polygona ſimilia dividuntur in ſimilia triangula.

Ducantur rectæ AC, AD, FH, FI. In triangulis P, S, anguli B & G (*per 1. def. h*) ſunt æquales, & BA: GF:: BC: GH. Ergo (*per 6. h*) P & S ſunt ſimilia: ergo (*per 1. def. h*) anguli BCA, GHF ſunt æquales: ergo his ſubductis ex angulis BCD, GHI (*per 1. def. h*) æqualibus, reſtant anguli ACD, FHI æquales. Dein (*per 4. h*) AC: FH:: BC: GH. Sed (*ex ſuppoſ.*) BC: GH:: CD: HI. Ergo (*per 11. l. 5.*) AC: FH:: CD: HI. Ergo (*per 6. h*) triangula Q & T ſunt ſimilia. Eodem modo oſtendas R & V, & quotquot fuerint, eſſe ſimilia.



PARS SECUNDA.

Polygona ſimilia ſunt ad ſe invicem in proportione duplicata laterum (AB, FG) *inter æquales angulos exiſtentium.*

Quia triangula P, S, & Q, T, & R, V, ſunt ſimilia, Ratio P ad S (*per 19. h*) eſt duplicata rationis AB ad FG; & Ratio Q ad T duplicata rationis CD ad HI; & Ratio R ad V duplicata rationis DE ad IK: ergo (*per 12. l. 5.*) Ratio omnium P, Q, R, ad omnia S, T, V, id eſt Polygoni ad Polygonum, eſt duplicata rationis laterum AB, CD, DE ad FG, HI, IK, id eſt (quia hæc latera ſupponuntur homologa) duplicata rationis AB ad FG.

PROPOSITIO XXIII.

In circulis æqualibus anguli, sive ad centrum, sive ad peripheriam sunt inter se ut arcus subtensi.

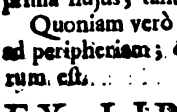
De angulis ad centrum demonstratur eodem modo quo prima hujus; tantùm loco Prop. 38. l. 1. citetur 29. l. 3.

Quoniam verò anguli ad centrum dupli sunt angulorum ad peripheriam; quod de illis ostenditur, de his etiam verum est.

EX LIBRO UNDECIMO.

Ex Libro hoc delegi, quæ ad Trigonometriam Sphæricam sunt maximè necessaria; adjecique nonnulla, quæ videbantur in Euclide desiderari.

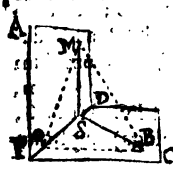
DEFINITIONES.



3. Linea (AB) dicitur *Rectus* seu *Perpendicularis plano* (DD) si cum omnibus lineis (BD) in plano ductis, à quibus tangitur, efficiat angulos rectos (ABD, ABD.)

Ut linea aliqua sciatur esse plano rectæ, satis est quòd cognoscatur esse perpendicularis duabus rectis in plano intersecantibus, ut perspicuum fiet *Prop. 4. h.*

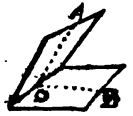
4. Planum est *Rectum* seu *Perpendiculare plano*, si omnes rectæ (CD) in uno plano (FB) ductæ perpendiculariter ad sectionem communem (AB) sint alterius plano (EB) perpendiculares. Ex hac definitione sequitur, quòd quandocunque planum plano infinitè perpendiculariter, non possit in eorum uno duci linea perpendicularis ad planorum sectionem communem, quin recta sit alteri plano: imò nec duci possit in uno plano aliqua (MS) perpendicularis lineæ (BS) ductæ in altero plano obliquè ad sectionem communem (FD), quin illa (MS) sit alteri plano (CD) perpendicularis.



Ducatur BO perpendiculariter ad sectionem communem; factâque MS æquali ipsi BO, ducantur etiam MO, MB.

Nunc in triangulis MSB, MOB, angulus MSB est (*ex suppos.*) rectus, sicut & (*per hanc & præced. def.*) angulus MOB; latus MB commune, & latera MS, BO (*ex constr.*) æqualia. Ergo (*per 4. Corol. p. 47. l. 1.*) tertium latus MO æquatur tertio BS. Ergo in triangulis MOS, BOS habentur MS & BO, item MO & BS æqualia, & tertium (seu distantia OS) commune: ergo (*per 8. l. 1.*) angulus MSO æquatur angulo BOS (*ex constr.*) recto: ergo MS est perpendicularis sectioni communi FD, adeoque & plano CD.

6. Plani ad planum inclinatio est angulus acutus (AOB,) quem efficiunt rectæ (AO, BO) in utroque plano ductæ perpendiculariter ad sectionem planorum communem.



PROPOSITIO I.

Non potest linea recta una pars esse in plano, altera extra idem planum.

Patet ex defin. lineæ rectæ & plani; nam utrumque ducitur per viam brevissimam, quæ unica est inter eosdem terminos.

PROPOSITIO II.

Omne triangulum in uno est plano, & duæ rectæ se mutuò secantes sunt in eodem plano.

1. *Pars.* Si trianguli CBE aliqua pars FEB non sit in eodem plano, quo altera CEF; rectæ CB una pars CF erit in plano CFE, alia FB extra idem planum. Sed hoc est contra 1. *h.* Ergo &c.

2. *Pars.* Quia triangulum CBE est in uno plano, rectæ CE, BE, adeoque (*per 1. h*) totæ CD, BA sunt in eodem plano.

Corollarium.

Ex eodem puncto (C) *non possunt erigi duæ* (CA, CR) *perpendiculares ad idem planum* [EN.]

Nam ducto per illas plano MN, anguli RCN, ACN (*per 3. def. h*) erunt recti, adeoque æquales, quod est absurdum.

PROPOSITIO III.

Si duo plana (AG, AP) *se mutuò secuerint, sectio communis* (AB) *erit linea recta.*

Si neges; à puncto A ad B duc in plano AG rectam AOB, & in plano AP rectam ASB; sic duæ rectæ AOB, ASB spatium includent; quod est absurdum.

PROPOSITIO IV.

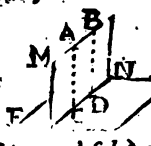
Si linea (AB) *sit recta duabus* (FP, GD) *se mutuò secantibus, est recta plano* (FGPD) *per ipsas ducto.*

Si neges; per B transeat planum RS, cui AB sit recta; & per lineas AB, BG ducatur aliud planum triangulare ARB secans planum RS alicubi in RB. Erit angulus ABR (*per 3. def. h*) rectus, adeoque æqualis angulo ABG (*ex suppos.*) recto; quod est absurdum.

PROPOSITIO VI.

Si linea (AC, BD) *sint eidem plano rectæ, sunt parallelæ.*

Super CD erigatur planum MN rectum plano EF. Jam si AC, BD non essent in plano MN, possent ex C & D erigi in plano MN aliæ perpendiculares plano EF; sicque ab eodem puncto possent excitari duæ perpendiculares eidem plano; quod est contra *Corol. p. 2. h.* Ergo sunt in plano MN. Cùm igitur anguli ACD, BDC sint (*per 3. def. h*) recti, lineæ AC, BD (*per 29. l. 1.*) sunt parallelæ.



PROPOSITIO VIII.

Si Parallelarum (AC, BD) *una* (BD) *sit recta plano* EF) *altera* (AC) *erit eidem recta.*

Super CD erigatur planum MN rectum plano EF. Quod BD, cùm sit (*ex suppos.*) recta plano EF, sit in plano MN,

oſtenditur ut in *Prop. 6. h.* Ergo cùm AC, BD ponantur parallelæ, etiam AC (*per 36. def. l. 1.*) erit in plano MN. Jam ſi AC non ſit recta plano EF, ex C erigatur CR in plano MN recta plano EF. Erunt (*per 6. h.*) CR, DB parallelæ: ergo (*per 30. l. 1.*) CR eſt etiam parallela ad AC: ergo duæ parallelæ CR, AC concurrunt in C; quod eſt abſurdum.

PROPOSITIO IX.

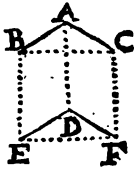
Rectæ (AB, EF) *ſi ſint eidem* (CD) *parallelæ, licèt non ſint omnes in eodem plano, ſunt inter ſe parallelæ.*

Rectas AB, EF, CD ſecet planum OKH, cui perpendicularis ſit CD: erunt (*per 8. h.*) etiam AB, EF eidem plano rectæ: ergo (*per 6. h.*) ſunt inter ſe parallelæ.

PROPOSITIO X.

Si duæ rectæ (AB, AC) *ſint duabus* (DE, DF) *parallelæ; utſi in planis diverſis, faciunt angulos* (A & D) *æquales.*

Fiant æquales AB & DE, AC & DF; & ducantur AD, CF, BE, CB, FE. Quia AB, DE ſunt æquales (*ex conſtr.*) & (*ex ſuppoſ.*) parallelæ, AD, BE ſunt (*per 33. l. 1.*) æquales & parallelæ: item quia AC, DF ſunt parallelæ & æquales, AD, CF ſunt (*per 33. l. 1.*) parallelæ & æquales: ergo BE, CF ſunt (*per ax. 1.*) æquales, & (*per 9. h.*) parallelæ: ergo (*per 33. l. 1.*) CB, FE ſunt parallelæ & æquales: ergo triangula BAC, EDF ſunt ſibi mutuò æquilatera: ergo (*per 8. l. 1.*) anguli BAC, EDF ſunt æquales.



PROPOSITIO XIII.

Linea ex eodem puncto ducta nequeunt amba ad idem plagium eſſe recta.

Vide Corol. p. 2. h.

PROPOSITIO XIV.

Si eadem recta (AG. Vide Fig. Prop. 15.) *ſit duobus planis* (BC, EF) *perpendicularis; plana ſunt parallela.*

Patet; nam planum unum BC non potest ad aliud EF inclinari, quin inclinetur ad rectam AG inter plana interceptam, quæ proinde non erit perpendicularis.

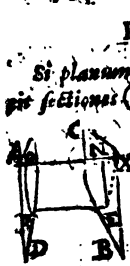
PROPOSITIO XV.

Si duæ rectæ (BA, GA) *se mutuò tangentes, ad totidem* (ED, FD) *se mutuò tangentes, sint parallelæ; plana per istas ductæ erunt parallela.*

Ex A ducatur AG recta plano EF, & ponantur GH, GI parallelæ ad DE, DF. Erunt (*per 9. h*) GH, GI etiam parallelæ ad AB, AC. Cùm igitur anguli IGA, HGA (*per 3. def. h*) sint recti, erunt (*per 27. l. 1.*) anguli CAG, BAG recti; ergo (*per 4. h*) AG est recta plano BC: sed (*ex constr.*) est etiam recta plano EF: ergo (*per 14. h*) plana BC, EF sunt parallela.

PROPOSITIO XVI.

Si planum (FONE) *secet plana parallela* (AD, CB,) *facit sectiones* (FO, EN) *parallelas.*



Nam si non sint parallelæ, cùm sint in eodem plano FONE, productæ alicubi concurrent, adeoque concurrent etiam plana parallela AD, CB, in quibus istæ lineæ FO, EN existunt; quod est absurdum.

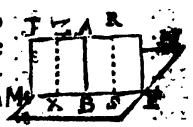
Corollarium.

Si duo plana (AD, CB) *fuerint parallela, & in eorum uno* (AD) *ducatur recta* (FO) *& huic* (FO) *altera parallela* (EN,) *cujus extremum* (E) *sit in alio plano* (CB;) *erit tota* (EN) *in hoc plano* (CB.)

Per rectas FO, EN, ducatur planum FONE, & producatur donec secet planum CB. Erit EN sectio planorum CB, FONE; adeoque parallela ad FO. Si neges; sit EX illa sectio: ergo (*per 16. h*) EX, FO sunt parallelæ. Sed (*ex supp.*) FO, EN sunt parallelæ: ergo (*per 30. l. 1.*) EX, EN, sunt parallelæ: quod est absurdum, cùm concurrant in E.

PROPOSITIO XVIII.

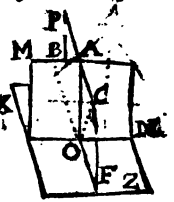
Si recta (AB) *ſit perpendicularis plano* (MN) *omnia plana per ipſam* (AB) *ducta ſunt iſti plano* (MN) *perpendicularia.*



Per AB tranſeat planum FP, in quo ducantur quotcunque ZX, RS parallelæ ad AB. Erunt ZX, RS (*per* 8. *h*) rectæ plano MN: ergo (*per* 4. *def. h*) planum FP eſt eidem rectum.

PROPOSITIO XIX.

Si plana (MN, PF) *ſe mutuò ſecantia ſint recta plano* (XZ) *eorum ſectio communis* (OA) *eſt eidem* (XZ) *perpendicularis.*

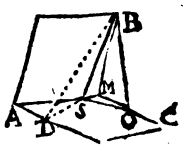


Si neges; ducatur ex O in plano MN recta OB, & in plano PF recta OC, perpendiculares ad XZ. Erunt (*per* 6. *h*) OB, OC parallelæ; quod eſt abſurdum, cùm ſe tangant in O.

Corollarium.

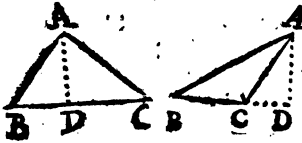
Si per duo plana (AB, AC) *ad ſe mutuò inclinata ducatur aliud* (BOS) *eorum uni* (AC) *rectum, ſecabit utrumque* (AB, AC) *vel neutrum, perpendiculariter ad eorum ſectionem communem* (AM.)

Si neges; ſecet planum AB in BS perpendiculariter, & AC in OS obliquè ad ſectionem communem AM. Ex S erigatur SD in plano AC perpendicularis ad AM. Jam quia AM eſt ad BS (*ex ſuppoſ.*) & ad SD (*ex conſtr.*) recta; eſt (*per* 4. *h*) recta plano BSD: ergo planum AC (*per* 18. *h*) eſt rectum plano BSD. Sed idem AC eſt [*ex ſuppoſ.*] rectum plano BOS: ergo planorum BSD, BOS ſectio communis BS eſt (*per* 19. *h*) recta plano AC: ergo (*per* 18. *h*) planum AB eſt rectum plano AC, ad quod ſupponitur inclinatum; quod eſt abſurdum. Ergo &c.



FINIS.

Pag.	Lin.	Errata.	Corrige.
12.	34. & 35.	AMC,	AMB.
18.	33.	Io,	Io, o:
29.	18.	*l.* 1.	*l.* 6.
34.	1.	& his	Ex his.
42.	2.	*Prop.* 9. 15.	*Omnia.*
44.	22.	concurrant.	concurrent.
52.	23.	(*per* 23. *l.* 1.)	(*per* 29. *l.* 3.)
68.	3.	(*per* 1, *c.* 1. *h*)	(*per* 1. *c.* 2. *h*)
69.	37.	(*per* 2. *Cor.*)	(*per* 3. *Cor.*)
132.	2.	BC in.	BC utcunque in
Ibid.	3.	EF.	BC
134.	3.	per E.	per D & E
150.	2.	(*p.* 17. *b*)	(*per* 16. *b*)
151.	16.	quantitatem.	quantitate,
Ibid.	21.	A =	A = B.



Prætereà hæ Figuræ apponi debuerant pag. 15. *Notando.* 2. cui Figuræ ibidem positæ non satis congruunt.